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## A GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES

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ARTICLE INFO	ABSTRACT
<i>Article History:</i> Received 6 <sup>th</sup> September, 2017 Received in revised form 25 <sup>th</sup> October, 2017 Accepted 4 <sup>th</sup> November, 2017 Published online 28 <sup>th</sup> December, 2017	In this paper, we introduce and study a new class of sets namely $\overline{g}$ -closed sets which settled in between the class of closed sets and the class of g-closed sets and then we study many basic properties of $\overline{g}$ -closed set together with the relationships of some other sets. As applications of $\overline{g}$ -closed sets, we introduce some new separation properties, namely $\overline{T}$
Key words:	introduce and study new types of continuity namely $\overline{g}$ -continuity and $\overline{g}$ -irresoluteness.
$\overline{g}$ -closed sets; $\overline{T}$ -spaces, $\overline{T}^*$ -spaces; $*\overline{T}$ -spaces, $\overline{g}$ -continuity.	

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## INTRODUCTION

The study of g-closed sets in a topological space was initiated by Levine [3] in 1970. Levine [1] also introduced the concept of semi-open sets and semi-continuity in topological spaces in 1963. Bhattacharya and Lahiri [7] introduced sg-closed sets in 1987. Arya and Nour [8] defined gs-closed sets in 1990. N. jasted [2] introduced the concepts of  $\alpha$ -closed sets for topological spaces in 1965. Maki *et al* generalized  $\alpha$ -closed sets to  $\alpha$ g-closed sets [13] and  $\alpha$ g-closed sets [15] in 1993 and 1994 respectively. Dontchev [16] (resp. Palaniappan and Rao [14], Gnanambal [18]) introduced gsp-closed (resp. rg-closed, gpr-closed) sets in 1995 (resp. 1993, 1997). Veera Kumar introduced  $\hat{g}$ -closed sets [22],  $\psi$ -closed sets [20], \*g-closed sets [25], g\*-closed sets [24] in 2007.

In this paper we study relationships of  $\overline{g}$  -closed sets with the above mentioned sets.

Norman Levine [3], Bhattacharya and Lehiri [7] and Devi *et al.* [12] introduced  $T_{1/2}$ -spaces, semi- $T_{1/2}$  spaces and  $T_b$  and  $T_d$ -spaces respectively. Devi *et al* [19] again introduce  $_{\alpha}T_b$ -spaces and  $_{\alpha}T_d$ -spaces. Veera Kumar introduced  $T_f$ -spaces [22],  $\hat{T}_b$ -spaces and  $_{\alpha}\hat{T}_b$ -spaces [23],  $T^*_{1/2}$ -spaces and  $*T_{1/2}$ -spaces [21]. Recently Manoj *et al.* [24] introduced  $\hat{T}_f$ -spaces.

\**Corresponding author:* Shailendra Singh Rathore Department of Mathematics, Nehru P. G. College, Chhibramau, Kannauj, U.P., India We introduce and study new classes of spaces, namely the class of  $\overline{T}$ -spaces, the class of  $\overline{T}^*$ -spaces and the class of

 $^{\ast}\overline{T}$  -spaces. Further we characterize and study some relationships of these spaces with the above defined spaces.

We also introduced  $\overline{g}$ -continuous maps,  $\overline{g}$ -irresolute maps and investigate some of their properties.

Throughout this paper  $(X, \tau)$ ,  $(Y, \sigma)$  and  $(Z, \eta)$  represent nonempty topological spaces on which no separation axioms are assumed unless or otherwise mentioned. For a subset A of a space  $(X, \tau)$ , cl(A), int(A), pcl(A) and A<sup>c</sup> denote the closure of A, the interior of A, pre-closure of A and the complement of A respectively.

### Preliminaries

We recall the following definitions which are useful in the sequel.

**Definition:** A subset A of a topological space  $(X, \tau)$  is called semi-open [1] (resp. semi-closed, pre-open [5], pre-closed,  $\alpha$ open [2],  $\alpha$ -closed, semi-preopen [6], semi-pre closed, regular open [28], regular closed) if A  $\subseteq$  cl(int(A)) (resp. int(cl(A))  $\subseteq$ A, A  $\subseteq$  int(cl(A)), cl(int(A))  $\subseteq$  A, A  $\subseteq$  int(cl(int(A))), cl(int(cl(A)))  $\subseteq$  A, A  $\subseteq$  cl(int(cl(A))), int(cl(int(A)))  $\subseteq$  A, A = int(cl(A)), A = cl(int(A))).

**Definition:** A subset A of a topological space  $(X, \tau)$  is called g-closed [3] (resp. sg-closed[7], gs-closed[8], ga-closed [13], ag-closed [15], rg-closed [14], gpr-closed [18], gsp-closed

[16],  $\hat{g}$ -closed [22],  $\psi$ -closed [20], \*g-closed [25],  $\hat{\hat{g}}$ -closed

[24], g\*-closed [21], \*gs-closed [26], #gs-closed [27]) if cl(A)  $\subseteq$  U (resp. scl(A)  $\subseteq$  U, scl(A)  $\subseteq$  U,  $\alpha$ cl(A)  $\subseteq$  U,  $\alpha$ cl(A)  $\subseteq$  U, cl(A)  $\subseteq$  U, pcl(A)  $\subseteq$  U, spcl(A)  $\subseteq$  U, cl(A)  $\subseteq$  U, scl(A)  $\subseteq$  U, cl(A)  $\subseteq$  U, cl(A)  $\subseteq$  U, cl(A)  $\subseteq$  U, scl(A)  $\subseteq$  U) whenever A  $\subseteq$  U and U is open (resp. semi-open,  $\alpha$ open, open, regular open, regular open, semi-open, sgopen,  $\hat{g}$ -open, sg-open,  $\hat{g}$ -open, \*g-open) in (X,  $\tau$ ).

The complement of a g-closed (resp.  $\hat{g}$ -closed,  $\hat{g}$ -closed, \*gclosed) set is called g-open (resp.  $\hat{g}$ -open,  $\hat{g}$ -open, \*g-open) set.

**Definition:** A function f:  $(X, \tau) \rightarrow (Y, \sigma)$  is called gcontinuous[10] (resp. sg-continuous[11], gs-continuous [17], ag-continuous [18], rg-continuous [14], gpr-continuous [18], gsp-continuous [16], \*g-continuous [23], g\*-continuous [21],  $\hat{g}$ -continuous [23],  $\hat{g}$ -continuous [24], semi-continuous[1], irresolute [4], gc-irresolute [10], sg-irresolute [11],  $\hat{g}$ irresolute [23],  $\hat{g}$ -irresolute [24], \*g-irresolute [25]) if f<sup>-1</sup> (V) is g-closed (resp. sg-closed, gs-closed, ag-closed, rg-closed, gpr-closed, gsp-closed, \*g-closed, g\*-closed,  $\hat{g}$ closed, semi-closed, semi-closed, gc-closed,  $\hat{g}$ closed,  $\hat{g}$ -closed, \*g-closed) set in (X,  $\tau$ ) for every closed (resp. closed, closed, closed, closed, closed, closed, closed, closed,  $\hat{g}$ -closed,  $\hat{g}$ -closed, semi-closed, semi-closed, semi-closed, semi-closed, semi-closed, semi-closed, closed, semi-closed, closed, semi-closed, semi-closed, semi-closed, semi-closed, semi-closed, semi-closed, semi-closed, semi-closed, closed, semi-closed, semi-closed,

**Definition:** A space (X,  $\tau$ ) is called T<sub>1/2</sub> [3] (resp. T<sub>b</sub> [12], T<sub>d</sub> [12],  $_{\alpha}T_{b}$  [19],  $_{\alpha}T_{d}$  [19], semi-T<sub>1/2</sub> [7], T<sub>f</sub> [22],  $\hat{T}_{b}$  [23], T\*<sub>1/2</sub> [21], \*T<sub>1/2</sub> [21],  $\hat{T}_{f}$  [24]) space if every g-closed (resp. gsclosed, gs-closed,  $\alpha$ g-closed,  $\alpha$ g-closed, sg-closed, g-closed, gs-closed, g\*-closed, g-closed, g-closed) set is a closed (resp. closed, g-closed, g-closed, semi-closed,  $\hat{g}$ -closed,  $\hat{g}$ closed, closed, g\*-closed,  $\hat{g}$ -closed) set in (Y,  $\sigma$ ).

## Basic properties of $\overline{\mathbf{g}}$ -closed sets

In this section we study the relationship of  $\overline{g}$  -closed sets with other sets.

**Definition:** A subset A of topological  $(X, \tau)$  is called  $\overline{g}$ -closed set if  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is a  $\hat{\hat{g}}$ -open set in  $(X, \tau)$ .

The complement of  $\overline{g}$  -closed set is called  $\overline{g}$  -open set.

## Theorem:

- 1. Every closed (or \*g-closed or g\*-closed or  $\hat{g}$  -closed) set is  $\overline{g}$  -closed set.
- 2. Every  $\overline{g}$ -closed set is ag-closed (or g-closed or rgclosed or gpr-closed or gs-closed) set.

Next examples show that converse of the above theorem is not true in general.

*Example:* Let  $X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{a, b\}, X\}$ . Consider  $A = \{a, c\}$  then A is not a closed set. However A is  $\overline{g}$  -closed set. The set  $B = \{b, c\}$  is  $\alpha g$ -closed, rg-closed, gpr-closed and gs-closed set. However B is not a  $\overline{g}$  -closed set.

**Example:** Let X = {a, b, c},  $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ . Consider A = {b} then A is not a \*g-closed set and g\*-closed set. However A is a  $\overline{g}$ -closed set. The set B = {a, b} is not a  $\hat{g}$ -closed set. However it is  $\overline{g}$ -closed set.

Therefore the class of  $\overline{g}$ -closed sets is properly contained in the class of g-closed sets, the class of  $\alpha g$ -closed sets, the class of rg-closed sets, the class of gpr-closed sets, the class of gs-closed sets. Also this new class is properly contains the class of closed sets, the class of \*g-closed sets, the class of g\*-closed sets, the class of g\*-closed sets, the class of g\*-closed sets.

**Remark:**  $\overline{g}$ -closed set is independent from semi-closed sets, sg-closed sets,  $\alpha$ -closed sets,  $\psi$ -closed sets, #gs-closed sets and \*gs-closed sets.

The following examples support the above results.

*Example:* Let X = {a, b, c},  $\tau = \{\phi, \{a\}, X\}$ . Consider A = {a, b}, then A is not a semi-closed set. However A is  $\overline{g}$ -closed set. The set B = {a, b} is a  $\overline{g}$ -closed set but not sg-closed,  $\psi$ -closed and #gs-closed sets.

**Example:** Let  $X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ . Consider  $A = \{a\}$  then A is not a  $\overline{g}$ -closed set. However A is semi-closed set. The set  $B = \{a\}$  is sg-closed,  $\psi$ -closed and #gs-closed set but not a  $\overline{g}$ -closed set.

**Example:** Let  $X = \{a, b, c\}, \tau = \{\phi, \{b\}, \{b, c\}, X\}$ . Consider  $A = \{c\}$  then A is not a  $\overline{g}$ -closed set. However A is  $\alpha$ -closed set. The set  $B = \{a, b\}$  is not  $\alpha$ -closed set. However B is  $\overline{g}$ -closed set.

*Example:* Let  $X = \{a, b, c\}, \tau = \{\phi, \{a\}, \{b, c\}, X\}$ . Consider  $A = \{a, b\}$  then A is  $\overline{g}$ -closed set but not \*gs-closed set.

The following diagram shows the relationships of  $\overline{g}$  -closed set with other sets.



**Theorem:** Intersection of two  $\overline{g}$  -closed sets is not necessarily  $\overline{g}$  -closed set.

The following example supports the above theorem.

*Example:* Let X = {a, b, c, d},  $\tau = \{\phi, \{a, b\}, X\}$ . Consider A = {a, c} and B = {a, d} then A and B are  $\overline{g}$ -closed sets. However their intersection {a} is not a  $\overline{g}$ -closed set.

**Theorem:** Union of two  $\overline{g}$ -closed sets is again a  $\overline{g}$ -closed sets.

**Theorem:** If A be a  $\overline{g}$ -closed set in a space (X,  $\tau$ ) and A  $\subseteq$  B  $\subseteq$  cl(A) then B is also a  $\overline{g}$ -closed set.

**Theorem:** A is  $\overline{g}$  -closed set of  $(X, \tau)$  if and only if cl(A) - A

does not contain any non-empty  $\hat{g}$  -closed set.

#### Applications of $\overline{\mathbf{g}}$ -closed sets

In this section we introduce the following definitions.

**Definition:** A topological space  $(X, \tau)$  is called T -space if every  $\overline{g}$  -closed set in it is closed.

**Definition:** A topological space  $(X, \tau)$  is called  $\overline{T}^*$ -space if every g-closed set in it is \*\*g-closed set.

*Theorem:* Every T<sub>b</sub>-space is T -space.

The converse of the above theorem is not true as it can be seen from the following example.

*Example:* In example (3.04), (X,  $\tau$ ) is  $\overline{T}$  -space but not  $T_b$ -space.

**Theorem:** Every  $\hat{T}_{b}$ -space is  $\overline{T}$ -space.

The converse of the above theorem is not true as it can be seen from the following example.

**Example:** In example (3.04), (X,  $\tau$ ) is  $\overline{T}$  -space but not  $\hat{T}_{b}$  -space.

**Theorem:** Every  $\overline{T}$  -space is  $\hat{\hat{T}}_{1/2}$  -space.

The converse of the above theorem is not true as it can be seen from the following example.

*Example:* In example (3.03), (X,  $\tau$ ) is  $\hat{T}_{1/2}$ -space but not  $\overline{T}$ -space.

**Theorem:** Every  $T_{1/2}$  -space ( $_{\alpha}T_{b}$ -space) is  $\overline{T}$  -space. **Theorem:** Every  $\hat{T}_{f}$  -space is  $\overline{T}^{*}$  -space.

The converse of the above theorem is not true as it can be seen from the following example.

*Example:* In example (3.03), (X,  $\tau$ ) is  $\overline{T}^*$ -space but not  $\hat{T}_f$ -space.

**Theorem:** Every  ${}^{*}T_{1/2}$ -space and so  ${}^{\alpha}T_{c}$ -space is  $\overline{T}^{*}$ -space.

The converse of the above theorem is not true as it can be seen from the following example.

*Example:* In example (3.03), (X,  $\tau$ ) is  $\overline{T}^*$ -space but not  ${}^*T_{1/2}$ -space.

**Theorem:** A topological space  $(X, \tau)$  is  $T_{1/2}$ -space iff it is  $\overline{T}$ -space and  $\overline{T}^*$ -space.

**Theorem:** Every  $\overline{T}$  -space is  $T^*_{1/2}$  -space.

The converse of the above theorem is not true as it can be seen from the following example.

*Example:* In example (3.02), (X,  $\tau$ ) is  $T^*_{1/2}$ -space but not T -space.

**Theorem:** Every  ${}_{\alpha}T_{b}$  -space is  $\overline{T}$  -space.

**Definition:** A space  $(X, \tau)$  is called  ${}^{*}\overline{T}$  -space if every  $\alpha$ g-closed set is  $\overline{g}$  -closed.

**Theorem:** Every  ${}_{\alpha}T_{b}$  -space is  ${}^{*}\overline{T}$  -space.

The converse of the above theorem is not true as it can be seen from the following example.

**Example:** In example (3.02), (X,  $\tau$ ) is  ${}^{*}\overline{T}$  -space but not a  ${}_{\alpha}T_{b}$  -space.

**Theorem:** Every  ${}^{*}\overline{T}$  -space is  $\overline{T}^{*}$  -space.

The converse of the above theorem is not true as it can be seen from the following example.

*Example:* In example (3.01), (X,  $\tau$ ) is  $\overline{T}^*$ -space but not a  $^*\overline{T}$ -space.

**Theorem:** Every  ${}^{*}\overline{T}$  -space is  ${}_{\alpha}T_{d}$  -space.

**Remark:**  $\overline{T}$  -space and  $*\overline{T}$  -space (or  $\overline{T}^*$  -space) are independent.

The following diagram shows the relationships between the separation axioms discussed in this section.



**Theorem:** If  $(X, \tau)$  is  ${}^{*}\overline{T}$  -space then for each  $x \in X$ ,  $\{x\}$  is either  $\alpha g$ -closed or \*\*g-closed.

**Theorem:** If  $(X, \tau)$  is  $\overline{T}^*$ -space then for each  $x \in X$ ,  $\{x\}$  is either closed or \*\*g-closed.

**Theorem:** A topological space  $(X, \tau)$  is T-space iff every singleton of X is either  $\hat{\hat{g}}$ -closed or open.

#### $\overline{\mathbf{g}}$ -continuous and $\overline{\mathbf{g}}$ -irresolute functions

In this section we introduce the following definitions.

**Definition:** A function  $f : (X, \tau) \to (Y, \sigma)$  is said to be  $\overline{g}$ -continuous if the inverse image of every  $\sigma$ -closed set in Y is  $\overline{g}$ -closed in X.

*Example:* Let  $X = \{a, b, c\}$  and  $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$  and  $\sigma = \{\phi, \{a\}, X\}$ . Define f:  $(X, \tau) \rightarrow (X, \sigma)$  by identity mapping then f is  $\overline{g}$ -continuous mapping.

**Definition:** A function  $f : (X, \tau) \to (Y, \sigma)$  is said to be  $\overline{g}$ -irresolute if the inverse image of every  $\overline{g}$ -closed set in Y is  $\overline{g}$ -closed in X.

*Example:* Let X = {a, b, c} and  $\tau = \{\phi, \{a\}, \{b, c\}, X\}$  and  $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ . Define f:  $(X, \tau) \rightarrow (X, \sigma)$  by identity mapping then f is  $\overline{g}$ -irresolute mapping.

### Theorem:

- 1. Every  $\overline{g}$  -irresolute map is  $\overline{g}$  -continuous map.
- 2. Every continuous map is  $\overline{g}$  -continuous map.
- 3. Every \*g-continuous (or g\*-continuous or  $\hat{g}$  continuous) map is  $\overline{g}$  -continuous map.
- 4. Every  $\overline{g}$ -continuous map is  $\alpha g$ -continuous (or rgcontinuous or gpr-continuous or gs-continuous) map.
- 5. Every  $\overline{g}$ -continuous map is g-continuous (or \*\*gs-continuous) map.

The converse of the above theorem is not true as it can be seen from the following example.

**Example:** The function f in example (5.01) is  $\overline{g}$ -continuous but not  $\overline{g}$ -irresolute.

*Example:* Let  $X = \{a, b, c\}, \tau = \{\phi, \{b\}, \{a, c\}, X\}$  and  $\sigma = \{\phi, \{a\}, X\}$ . Define f:  $(X, \tau) \rightarrow (X, \sigma)$  by f(a) = a, f(b) = b and f(c) = c then f is  $\overline{g}$  -continuous but not continuous.

*Example:* Let X = {a, b, c},  $\tau = \{\phi, \{a\}, \{b, c\}, X\}$  and  $\sigma = \{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$ . Define f: (X,  $\tau$ )  $\rightarrow$  (X,  $\sigma$ ) by f(a) = b, f(b) = a and f(c) = c then f is  $\overline{g}$ -continuous but not \*g-

continuous and  $\,\hat{g}$  -continuous.

*Example:* Let X = {a, b, c},  $\tau = \{\phi, \{a\}, \{a, b\}, X\}$  and  $\sigma = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$ . Define f: (X,  $\tau$ )  $\rightarrow$  (X,  $\sigma$ ) by identity map then f is  $\alpha$ g-continuous, rg-continuous, gpr-continuous, gs-continuous and \*\*gs-continuous but not  $\overline{g}$ -continuous.

Therefore the class of  $\overline{g}$  -continuous maps properly contains the class of continuous maps, the class of \*g-continuous maps,

the class of g\*-continuous maps and the class of  $\hat{g}$ -continuous maps and it is properly contained in the class of g-continuous maps, the class of  $\alpha g$ -continuous maps, the class of rg-continuous maps, the class of gpr-continuous maps, the class of gs-continuous maps.

**Remarks:** The composition of two  $\overline{g}$ -continuous function need not be  $\overline{g}$ -continuous again. For consider the following example.

**Example:** Let X = {a, b, c},  $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ ,  $\sigma = \{\phi, \{b\}, X\}$  and  $\eta = \{\phi, \{a\}, \{b, c\}, X\}$ . Define f: (X,  $\tau \to (X, \sigma)$  by f(a) = c, f(b) = b and f(c) = a. Define g : (X,  $\sigma \to (X, \eta)$  by g(a) = c, g(b) = b and g(c) = a. then f and g are  $\overline{g}$ -continuous for {a} is closed in (X,  $\eta$ ) but (gof)<sup>-1</sup>({a}) = f<sup>-1</sup>(g<sup>-1</sup>({a})) = f<sup>-1</sup>({c}) = {a} is not a  $\overline{g}$ -closed set in (X,  $\tau$ ). Hence gof is not a  $\overline{g}$ -continuous.

**Theorem:** The composition of two  $\overline{g}$ -irresolute function is again  $\overline{g}$ -irresolute.

**Theorem:** Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be an irresolute and closed then A is  $\overline{g}$ -closed in  $(X, \tau)$  implies f(A) is  $\overline{g}$ -closed in  $(Y, \sigma)$ .

**Remarks:**  $\overline{g}$ -continuity and semi-continuity are independent as seen from the following examples.

*Example:* Let  $X = \{a, b, c\}, \tau = \{\phi, \{a\}, X\}$  and  $\sigma = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ . Define f:  $(X, \tau) \rightarrow (X, \sigma)$  by f(a) = a, f(b) = c and f(c) = b. Then f is not a semi-continuous. However f is  $\overline{g}$ -continuous.

*Example:* Let X = {a, b, c},  $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$  and  $\sigma = \{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$ . Define f: (X,  $\tau$ )  $\rightarrow$  (X,  $\sigma$ ) by identity mapping. Then f is not a  $\overline{g}$ -continuous. However f is semi-continuous.

**Remarks:**  $\overline{g}$ -continuity and sg-continuity (or  $\psi$ -continuity or #gs-continuity) are independent as seen from the following examples.

*Example:* Let X = {a, b, c},  $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ and  $\sigma = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, X\}$ . Define f: (X,  $\tau$ )  $\rightarrow$  (X,  $\sigma$ ) by identity mapping then f is not a  $\overline{g}$ -continuous, however f is sg-continuous (or  $\psi$ -continuous or #gs-continuous).

*Example:* Let X = {a, b, c},  $\tau = \{\phi, \{a\}, X\}$  and  $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ . Define f:  $(X, \tau) \rightarrow (X, \sigma)$  by identity mapping. Then f is  $\overline{g}$ -continuous but not sg-continuous (or  $\psi$ -continuous or #gs-continuous).

**Theorem:** If f:  $(X, \tau) \rightarrow (Y, \sigma)$  is  $\overline{g}$  -continuous and Y is  $T_u$  – space then f is  $\overline{g}$  -irresolute.

**Theorem:** Let  $(X, \tau)$ ,  $(Y, \sigma)$  and  $(Z, \eta)$  be any three topological spaces. Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  be any two functions then gof:  $(X, \tau) \rightarrow (Z, \eta)$  is

- 1.  $\overline{g}$ -continuous if g is continuous and f is  $\overline{g}$ -continuous.
- 2.  $\overline{g}$  -irresolute if g is  $\overline{g}$  -irresolute and f is  $\overline{g}$  -irresolute.
- 3.  $\overline{g}$ -continuous if g is  $\overline{g}$ -continuous and f is  $\overline{g}$ -irresolute.
- 4. g-continuous if g is  $\overline{g}$ -continuous and f is gc-irresolute.

**Theorem:** Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be a  $\overline{g}$ -continuous map. If  $(X, \tau)$  is  $T_{u}$ -space then f is continuous.

**Theorem:** Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be an  $\alpha$ g-continuous map. If  $(X, \tau)$  is  ${}_{\alpha}T_{u}$ -space then f is  $\overline{g}$ -continuous.

**Theorem:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a g-continuous map. If  $(X, \tau)$  is  $T_u^*$ -space then f is  $\overline{g}$ -continuous.

**Theorem:** Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be onto,  $\overline{g}$ -irresolute and closed map. If  $(X, \tau)$  is  $T_u$ -space then  $(Y, \sigma)$  is also a  $T_u$ -space.

**Theorem:** Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be  $\hat{g}$ -irresolute and closed map. Then f(A) is a  $\overline{g}$ -closed set of  $(Y, \sigma)$  for every  $\overline{g}$ -closed set A of  $(X, \tau)$ .

**Theorem:** Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be onto, gc-irresolute and pre- $\overline{g}$ -closed map. If  $(X, \tau)$  is  $\overline{T}^*$ -space then  $(Y, \sigma)$  is also a  $\overline{T}^*$ -space.

**Theorem:** Let f:  $(X, \tau) \rightarrow (Y, \sigma)$  be onto,  $\alpha g$ -irresolute and pre- $\overline{g}$ -closed map. If  $(X, \tau)$  is  ${}^{*}\overline{T}$ -space then  $(Y, \sigma)$  is also a  ${}^{*}\overline{T}$ -space.

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