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Resarch Article

MAGNETOHYDRODYNAMIC CASSON FLUID FLOW WITH HALL EFFECT AND ROTATION IN THE PRESENCE OF AN INCLINED MAGNETIC FIELD

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A B S T R A C T

The main objective of the paper is to investigate the Casson fluid flow problem including Hall effect and Rotation in the presence of inclined magnetic field. Analytical solution has been found depending on the physical parameters including the Hall parameter, Hartmann number, Casson fluid parameter and the Rotation parameter. The influence of these parameters on velocity profiles are demonstrated graphically and the results are discussed. Further, it is observed that the primary and secondary velocity components retards when Hall parameter, Casson fluid parameter and Rotation parameter are increased. The increase in Hartmann number resists the primary velocity and accelerates secondary velocity but this is converse for increasing values of angle of inclination.

INTRODUCTION

The study of Casson fluid flow has been carried out by several authors. During the last decades extensive research work has been done on the fluid dynamics of biological fluids in the presence of magnetic field. For multiple reasons, applications of magnetohydrodynamics in physiological flow problems are of growing interest.

Puri and Kuishrestha [1] studied the unsteady hydromagnetic boundary layer in a rotating medium. Tokis and Geroyannis [2] further they analyzed the effects of fluctuation of the external stream on unsteady three dimensional flows of a viscous, electrically conducting and rotating fluid near a horizontal plate in the presence of transverse magnetic field.

Chaturvedi [3] studied effects of magnetic field and exponentially varying suction on the flow of incompressible viscous electrically conducting fluid past an impulsive started infinite plate. The Hall effects on MHD flow past an accelerated plate was examined by Deka [4]. Magnetohydrodynamic problem with Hall effect was analysed by Haythem Sulieman [5]. Guchhait et al [6] investigated the combined effect of Hall current and rotation on MHD flow in a rotaing vertical channel.

In real situation always the magnetic field cannot be horizontal or vertical to the axis of rotation and plate.

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In many applications like MHD generators, astrophysics, nuclear power reactors etc., the applied magnetic field may act obliquely to the flow. It is worth-mentioning that MHD is now undergoing a stage of great enlargement and differentiation of subject matter with inclined magnetic field. In this chapter an attempt is made to study the effect of MHD with Hall effect and rotation in the presence of an inclined magnetic field in a Casson fluid flow.

Formulation of the problem

Consider the flow of an incompressible electrically conducting, viscous fluid past an infinite and insulated flat plate occupying the plane $y = 0$. Initially the fluid and the plate rotate in unison with a uniform angular velocity $\bar{\Omega}$ about the y - axis normal to the plate. The x-axis is taken in the direction of the motion of the plate and z – axis lying on the plate normal to both x and y – axis. Relative to the rotating fluid, the plate is impulsively started from rest and set into motion with uniform acceleration in its own plane along the x - axis. An uniform magnetic field H_0 is applied in a direction which makes an angle θ with the positive direction of y - axis in the $xy -$ plane. Here $\bar{q}=(u,0,w)$ represents the velocity vector, $\bar{H}=(H_0 \sin\theta, H_0 \cos\theta, 0)$ is the magnetic induction, $\bar{E}=(E_x, 0, E_z)$ is the electrostatic field and $\bar{\Omega}=(0,\Omega_y,0)$ denotes uniform angular velocity.

The rheological equation of state for an isotropic and incompressible flow of a Casson fluid is as follows [7]

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \dfrac{P_y}{\sqrt{2\pi}}\right) e_{ij}, & \pi > \pi_c \\ 2\left(\mu_B + \dfrac{P_y}{\sqrt{2\pi_c}}\right) e_{ij}, & \pi < \pi_c \end{cases}$$

where $\pi = e_{ij}e_{ij}$ and e_{ij} are the $(i,j)th$ component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_B is plastic dynamic viscosity of the non-Newtonian fluid and P_y is the yield stress of the fluid. Governing equations are:

$$\nabla.\bar{q}=0 \tag{4.1}$$

$$\frac{\partial \bar{q}}{\partial t} + (\bar{q}.\nabla)\bar{q} + 2\bar{\Omega} \times \bar{q} = -\frac{1}{\rho}\nabla P + \nu\left(1+\frac{1}{\gamma}\right)\nabla^2\bar{q} + \frac{1}{\rho}(\bar{J} \times \bar{B}) \tag{4.2}$$

$$\frac{\bar{J}}{\sigma} = (\bar{E} + \bar{q} \times \bar{B}) - \frac{\bar{J}\times\bar{B}}{n.e} \tag{4.4}$$

where σ is the electrical conductivity, $\bar{J}$ is the current density, ρ is density, ν is kinematic viscosity, e is electric charge, n is the electron number density and γ is the Casson fluid parameter. The initial and boundary conditions are

$u = 0,\ w = 0$ for all $t \le 0$ and for all y
$u = U_0,\ w = 0$ for all $t > 0$ and $y = 0,$
$u \to 0,\ w \to 0$ for all $t > 0$ and $y \to \infty$
(4)

Physical quantities are cast in non-dimensional form by using the following non-dimensional scheme.

$$y^* = \frac{U_0 y}{\nu},\quad u^* = \frac{u}{U_0},\quad w^* = \frac{w}{U_0},\quad t^* = \frac{U_0^2 t}{\nu} \tag{5}$$

Now introducing the above non-dimensional quantities in equation (2), the components are

$$\frac{\partial u}{\partial t} = \left(1+\frac{1}{\gamma}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma H_0^2 \nu}{\rho U_0^2(1+\omega^2\tau^2)}(u + \omega\tau w) - \frac{2\nu}{U_0^2} w\Omega_y \tag{6}$$

$$\frac{\partial w}{\partial t} = \left(1+\frac{1}{\gamma}\right)\frac{\partial^2 w}{\partial y^2} - \frac{\sigma H_0^2 \nu}{\rho U_0^2(1+\omega^2\tau^2)}(\omega\tau u - w) + \frac{2\nu}{U_0^2} u\Omega_y \tag{7}$$

where the non-dimensional parameters are defined below

$M^2 = \frac{\sigma H_0^2 \nu}{\rho U_0^2}$ is the square of the Hartmann number,

$m = \omega\tau$ is the Hall Parameter

$K^2 = \frac{\nu\Omega_y}{U_0^2}$ is the Rotation parameter i.e., the reciprocal of Ekmann number and

The corresponding initial and boundary conditions (4) in non-dimensional forms are

$t \le 0 : u(y,t) = 0;\ w(y,t) = 0$ for all y.
$t > 0 : u(0,t) = 1, w(0,t) = 0$
$t > 0 : u(y,t) \to 0, w(y,t) \to 0$ as $y \to \infty$ (8)

Solution of the Problem

By introducing $q = u + iw$, equation (6) and (7) becomes

$$\frac{\partial q}{\partial t} = c\frac{\partial^2 q}{\partial y^2} - \left[\left(\frac{M^2\cos^2\theta}{1+m^2\cos^2\theta}\right)(1 - im\cos\theta) - 2iK^2\right] q \tag{9}$$

where $c = \left(1+\frac{1}{\gamma}\right)$

The initial and boundary conditions take the form

$$q(y,0) = 0,\ q(0,t) = 1,\ \ q(y,t) \to 0 \text{ as } y \to \infty \tag{10}$$

Using the abbreviation $\alpha = \left[\left(\frac{M^2\cos^2\theta}{1+m^2\cos^2\theta}\right)(1 - im\cos\theta) - 2iK^2\right]$,

Equation (2.9) can be written as

$$\frac{\partial q}{\partial t} = c\frac{\partial^2 q}{\partial y^2} - \alpha q \tag{11}$$

Also substitute $q(y,t) = e^{i\xi t} g(y)$ in (11), we have

$$c\, g''(y) - (i\xi + \alpha)g(y) = 0 \tag{12}$$

Equation (12) can be solved under the boundary conditions,

$$g(0) = e^{-i\xi t}, g(\infty) = 0 \tag{13}$$

The solution is

$$g(y) = e^{-i\xi t} e^{\frac{-y}{\sqrt{c}}\sqrt{i\xi+\alpha}} \tag{14}$$

Hence $q(y,t) = e^{i\xi t}\left[e^{-i\xi t} e^{-y\sqrt{i\xi+\alpha}}\right]$ (15)

Real and imaginary parts of equation (15) are

$$u(y,t) = e^{-yS_1}\cos yS_2 \tag{16}$$

$$w(y,t) = -e^{-yS_1}\sin yS_2 \tag{17}$$

where $a = \frac{M^2\cos^2\theta}{1+m^2\cos^2\theta}$; $b = -\frac{M^2 m\cos^3\theta}{1+m^2\cos^2\theta} - 2K^2$;

$S_1 = \frac{1}{\sqrt{c}}\sqrt{\frac{a+\sqrt{a^2+(\xi+b)^2}}{2}}$; $S_2 = \frac{1}{\sqrt{c}}\sqrt{\frac{-a+\sqrt{a^2+(\xi+b)^2}}{2}}$

RESULTS AND DISCUSSION

The effect of the Hall parameter m, the Hartmann number M^2, the angle of inclination θ, the rotation parameter K^2 and Casson fluid parameter in the velocity components u and w are illustrated in the figures for the clear understanding of the problem.

Figure 1 and 2 show that the primary and secondary velocity for various values of Hartmann number. Application of a transverse magnetic field to an electrically conducting fluid gives rise to a resistive type force called the Lorentz force. This force has the tendency to slow down the motion of the fluid in the boundary layer.

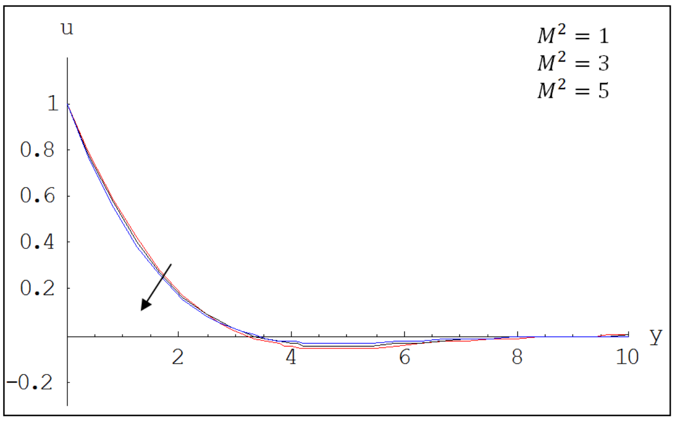

Figure 1 Effect of Hartmann number (M^2) on primary velocity profile when $\gamma = 0.2$; $m = 1$; $K^2 = 2$; $\theta = 30°$; $\xi = 1$; $K_p = 1$

It is clear from the above figures that Primary velocity decreases and secondary velocity increases with increase of the strength of the magnetic effect. The influence of Hall parameter on velocity profiles are depicted in the figures 3 and

4. An increase in Hall current leads to an decrease in primary and secondary velocity profiles. This is due to the fact that the effective conductivity decreases with the increase in Hall parameter which reduces the magnetic damping force.

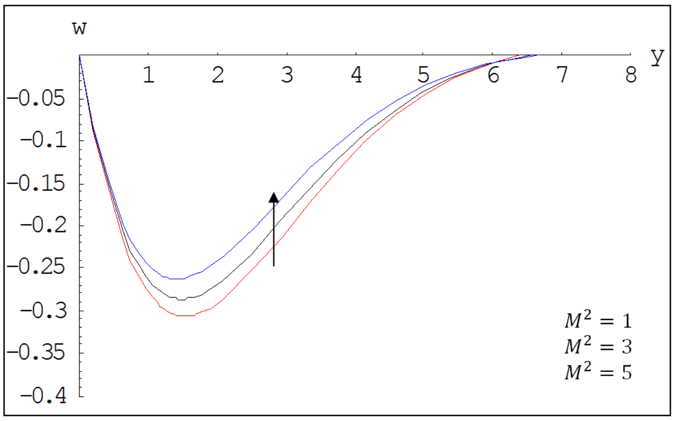

Figure 2 Effect of Hartmann number (M^2) on secondary velocity profile when $\gamma = 0.2$; $m = 1$; $K^2 = 2$; $\theta = 30°$; $\xi = 1$; $K_p = 1$

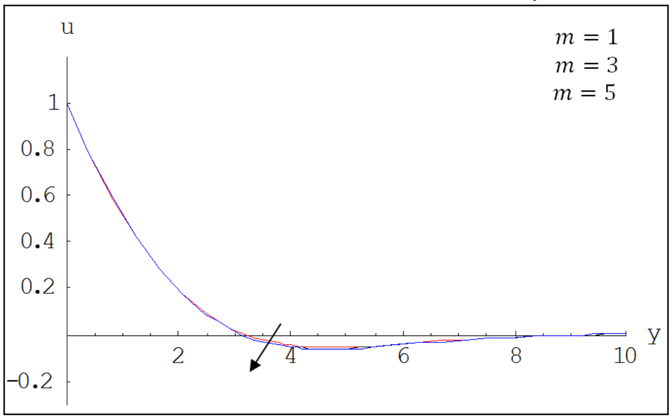

Figure 3 Effect of Hall parameter (m) on primary velocity profile when $\gamma = 0.2$; $M^2 = 1$; $K^2 = 2$; $\theta = 30°$; $\xi = 1$; $K_p = 1$

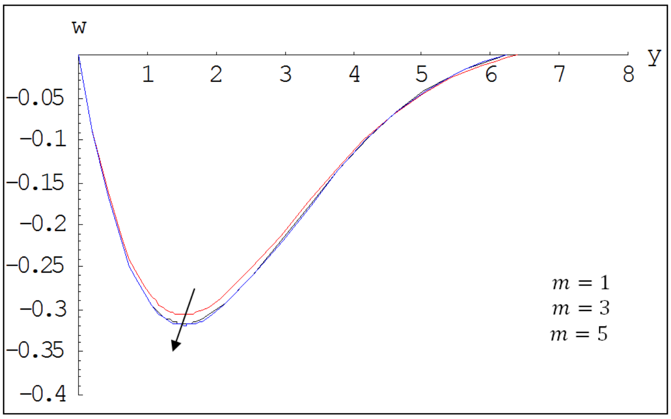

Figure 4 Effect of Hall parameter (m) on secondary velocity profile when $\gamma = 0.2$; $M^2 = 1$; $K^2 = 2$; $\theta = 30°$; $\xi = 1$; $K_p = 1$

The velocity profiles for different values of Casson fluid parameter and Rotation parameter are shown in the figures 5, 6 and 7, 8 respectively. It is observed that the primary and secondary velocity decreases with increasing values of the Casson fluid parameter and Rotation parameter. The Primary velocity increase and secondary velocity decrease for increasing values of angle of inclination are clearly shown in the figures 9 and 10

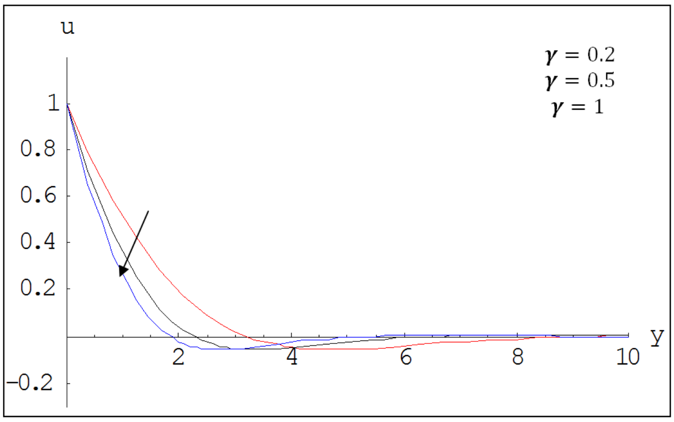

Figure 5 Effect of Casson fluid parameter (γ) on primary velocity profile when m = 1; $M^2 = 1$; $K^2 = 2$; $\theta = 30°$; $\xi = 1$; $K_p = 1$

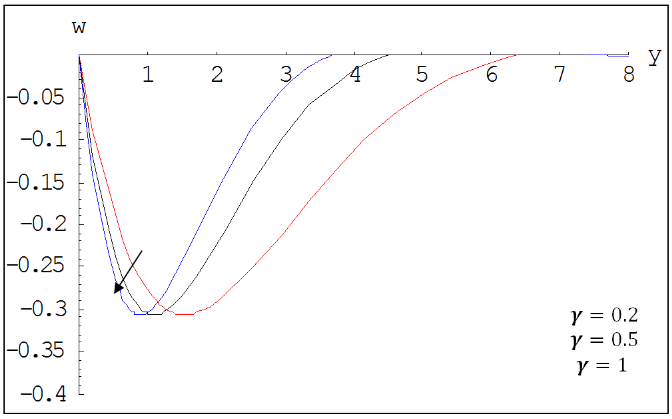

Figure 6 Effect of Casson fluid parameter (γ) on secondary velocity profile when m = 1; $M^2 = 1$; $K^2 = 2$; $\theta = 30°$; $\xi = 1$; $K_p = 1$

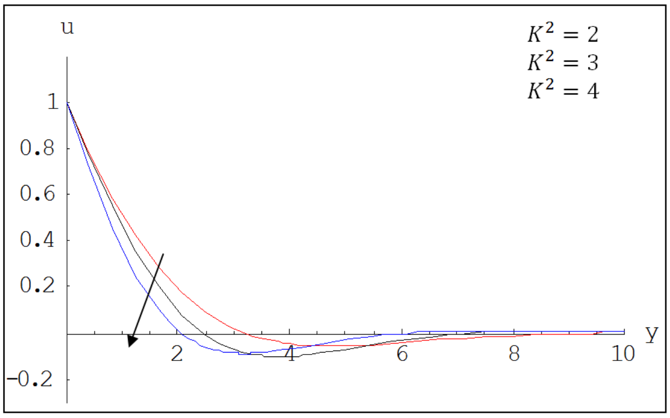

Figure 7 Effect of Rotation parameter (K^2) on primary velocity profile when $\gamma = 0.2$; $M^2 = 1$; $m = 1$; $\theta = 30°$; $\xi = 1$; $K_p = 1$

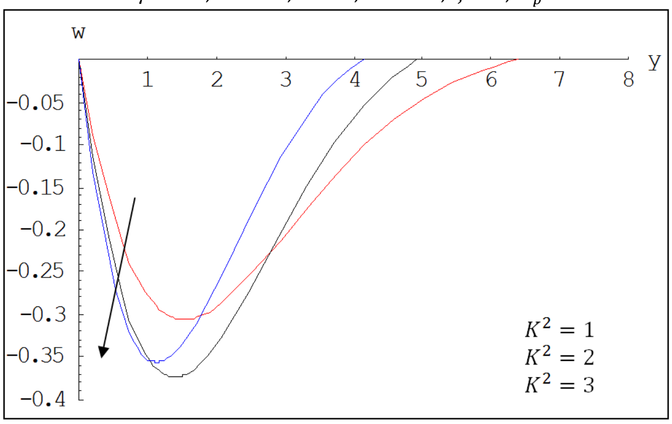

Figure 8 Effect of Rotation parameter (K^2) on secondary velocity profile when $\gamma = 0.2$; $M^2 = 1$; $m = 1$; $\theta = 30°$; $\xi = 1$; $K_p = 1$

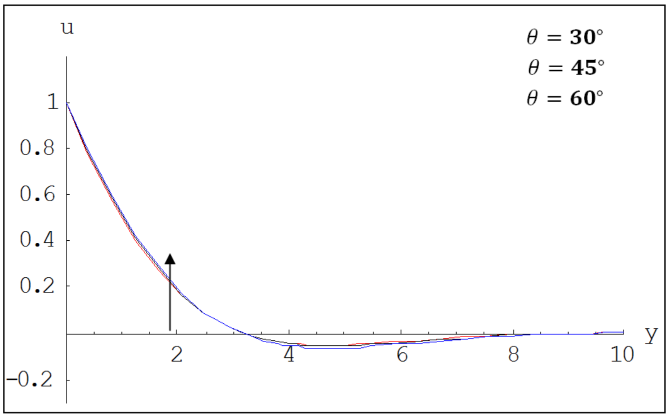

Figure 9 Effect of angle of inclination (θ) on primary velocity profile when $\gamma = 0.2;\ M^2 = 1; m = 1; K^2 = 2;\ \xi = 1;\ K_p = 1$

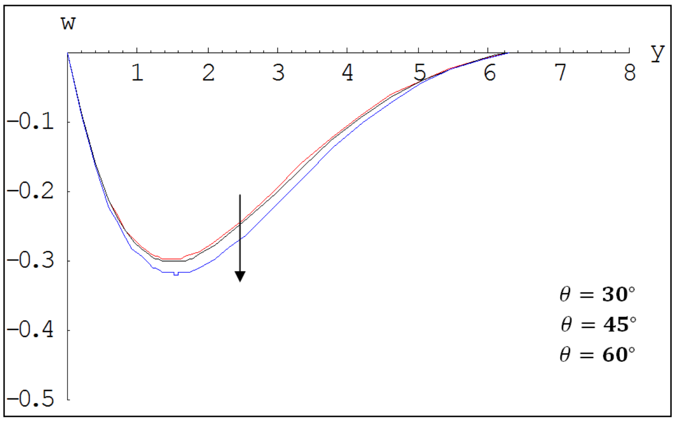

Figure 10 Effect of angle of inclination (θ) on secondary velocity profile when $\gamma = 0.2;\ M^2 = 1; m = 1; K^2 = 2;\ \xi = 1;\ K_p = 1$

CONCLUSION

From the above results and discussions it is concluded that, the primary and secondary velocity components retards when Hall parameter, Casson fluid parameter and Rotation parameter are increased. The increase in Hartmann number resists the primary velocity and accelerates secondary velocity but this is converse for increasing values of angle of inclination.

Reference

1. Puri, P. and Kuishrestha, P.K., 1976. Unsteady Hydromagnetic Boundary Layer in a Rotating Medium, Trans. *ASME. J. Appl. Mech*, 43: 205-208.
2. Tokis, J.N. and Geroyannis, V.S., 1981. Unsteady hydromagnetic rotating flow near an oscillating plate. Astrophysics and Space Science, 75, 393-405.
3. Chaturvedi, N., 1996. Energy Convers. Management, 37(5): 623-627.
4. Deka, R.K., 2008. Hall effects on MHD flow past an accelerated plate, Theoret. *Appl. Mech*., 35(4): 333-346.
5. Haythem Sulieman and Naji A. Qatanani, 2012. Magnetohydrodynamic Rayleigh Problem with Hall Effect, *IJMER*, 2(1): 390-402.
6. Guchhait, S.K., Das, S. and Jana, R.N., 2012. Combined effect of Hall current and Rotation on MHD mixed convection oscillating flow in a rotating vertical channel, *International journal of computer applications*, 49(13): 1-11.
7. Casson, N, 1959. A flow equation for Pigment –oil suspension of the priniting ink type. In: Mill, C.C., Ed., Rheology of disperse systems, Pergamon press, Oxford, 84-104.

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