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Research Article

WEIGHTED INTERPOLATION PROCESS ON ROOTS OF HERMITE POLYNOMIAL

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A R T I C L E I N F O

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A B S T R A C T

In this paper, we study the explicit representation and convergence of weighted (1; 0, 2)-interpolation on infinite interval, which means to determine a polynomial of degree $\leq 3n-1$ when the function value are prescribed at the zeroes of $H_n(x)$ and its derivative $H_n^{'}(x)$ respectively with certain conditions where $H_n(x)$ stands for Hermite polynomial.

INTRODUCTION

J. Balázs and P.Turan[1] introduced weighted (0, 2) interpolation on the zeroes of n^{th} ultraspherical polynomial $P^{(\alpha)}(x)$ (α > -1) with weight function f(x) = $(1-x^2)^{\frac{\alpha+1}{2}}$. He proved generally there does not exist any polynomial of degree ≤ 2n-1 satisfying the condition.

$$R_n(x_{i,n}) = \alpha_{i,n} \; (\rho R_n)''(y_{i.n)}) = \beta_{i,n}$$

where $\alpha_{i,n}$ and $\beta_{i,n}$ are given real numbers under the condition

$$R_n(0) = \sum_{i=1}^{n} \alpha_{i,n} l_{i,n}^2(0),$$

where $l_{i,n}$ represent the Lagrange – fundamental polynomial, corresponding to nodal point $x_{i,n}$ he proved that there exist a unique polynomial of degree ≤ 2n where n is even (if n is odd then uniqueness fails). He gave the explicit form of this polynomial and proved the convergence theorem.His study was extended by L. szili[5], he considered the roots as the zeroes of n^{th} hermite polynomial $H_n(x)$ for n even.

In another paper Srivastava and Mathur[7] study the mixed type weighted lacunary(0; 0 2) – interpolation on zeroes of $H_n(x)$ and $H_n^{'}(x)$ .P. Mathur and S. Dutta[8]alsostudied mixed type (0 2; 0) weighted lacunary interpolation onzeroes of $H_n(x)$ and $H_n^{'}(x)$ Let theinterscaled system of nodal points,

$$-\infty < x_1 < y_1 < x_2 < y_2 < ..... < x_{n-1} < y_{n-1} < x_n < +\infty$$

where $\{x_k\}_{k=1}^{n}$ and $\{y_k\}_{k=1}^{n-1}$ be zeroes of $H_n(x)$ and $H_n^{'}(x)$ where

$$H_n(x) = (-1)^n e^{x^2} \{e^{-x^2}\}^{(n)} \tag{1}$$

the fundamental polynomial of Lagrange – interpolation are given by,

$$l_k(x) = \frac{H_n(x)}{H_n^{'}(x)(x-x_k)}, \; k = 1,...., \; n \tag{2}$$

$$L_k(x) = \frac{H_n^{'}(x)}{H_n^{''}(x)(x-y_k)}, \; k = 1,...., \; n-1 \tag{3}$$

The problem- In this paper we study the following weighted $(1;0,2)$ interpolation –Let n be even then for given arbitrary numbers $\{g_k\}_{k=1}^{n-1}, \{g_k^{*}\}_{k=1}^{n}, \{g_k^{**}\}_{k=1}^{n}$, there exist a unique polynomial $R_n(x)$ of degree $\leq 3n-1$ such that,

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$$\begin{cases} R_n'(y_k) = g_k, k = 1,\dots, n-1 \\ R_n(x_k) = g_k^*, k = 1,\dots, n \\ \left(e^{-\frac{x^2}{2}} R_n(x)\right)''_{x=x_k} = g_k^{**}, k = 1,\dots, n \end{cases} \quad (4)$$

and $R_n'(0) = \sum_{k=1}^{n-1} \frac{g_k L_k^2(0) H_n'(0)}{y_k H_n(y_k)}$

For n odd, $R_n(x)$ does not exist uniquely. Precisely we shall prove the following-

Theorem 1. For n even,

$$R_n(x) = \sum_{n=1}^{n-1} g_k A_k(x) + \sum_{n=1}^{n} g_k^* B_k(x) + \sum_{n=1}^{n} g_k^{**} C_k(x) \quad (5)$$

Such that

$$R_n'(0) = \sum_{k=1}^{n-1} \frac{g_k L_k^2(0) H_n'(0)}{y_k H_n(y_k)}$$

Where $A_k(x), k = 1,\dots, n-1$ and $B_k(x)$, $k = 1,\dots, n$ are the fundamental polynomial of first kind and, $C_k(x)$, $k = 1,\dots, n$ are the fundamental polynomial of second kind of weighted $(1;0,2)$ - interpolation. Each such fundamental polynomial of degree at most $3n-1$ is given by

$$A_k(x) = \frac{L_k^2(x) H_n(x)}{2y_k H_n(y_k)} - \frac{H_n(x)}{2y_k H_n(y_k)} \int_0^x L_k'(t) L_k(t) dt + B_k(x) \quad (6)$$

$$B_k(x) = \frac{H_n'^2(x) l_k(x)}{H_n'^2(x_k)} - \frac{H_n(x)}{2H_n'^2(x_k)} \int_0^x \{6tl_k'(t) + l_k''(t)\} H_n'(t) dt - e^{-\frac{x_k^2}{2}} (9x_k^2 - 4n + 3) C_k(x) \quad (7)$$

$$C_k(x) = \frac{e^{\frac{x_k^2}{2}}}{2H_n'^2(x_k)} H_n(x) \int_0^x l_k(t) H_n'(t) dt \quad (8)$$

Where $l_k(x)$ and $L_k(x)$ are given by (1.2) and (1.3) respectively.

Theorem 2. Let the interpolated continuous function $f : R \to R$ is continuously differentiable, $f(0) = 0$ such that

$$\lim_{|x|\to\infty} x^{2r} f(x) e^{-\frac{x^2}{2}} = 0, r = 0,1,2,3,\dots$$
$$and \quad (9)$$
$$\lim_{|x|\to\infty} f'(x) e^{-\frac{x^2}{2}} = 0$$

Further, let the numbers δ_k satisfy the condition

$$\delta_k = O\left(\sqrt{n} e^{\delta y_k^2} \omega\left(f'; \frac{1}{\sqrt{n}}\right)\right), \quad (10)$$

$k = 1,\dots, n-1$

and $0 < \delta < 1$, such that

$R_n'(f,0) = \sum_{k=1}^{n-1} \frac{f(x_k) L_k^2(0) H_n'(0)}{y_k H_n(y_k)}$, satisfy the relation

$$e^{-\nu x^2} |R_n(f,x) - f(x)| = O(\sqrt{n})\ \omega(f'; \tfrac{1}{\sqrt{n}}), \nu > 1 \quad (11)$$

Which holds on whole real line and O does not depend on x and n.

Remark

$\omega(f,\delta)$ denotes the special form of modulus of continuity introduced by G. Freud given by

$$\omega(f,\delta) = \sup_{0\le t\le\delta} \|\rho(x+t) f(x+t) - \rho(x) f(x)\| + \|\tau(\delta x)\rho(x) f(x)\| \quad (12)$$

Where $\tau(x) = \begin{cases} |x|, & for \ |x| \le 1 \\ 1, & for \ |x| > 1 \end{cases}$

and $\|.\|$ denotes the supremum norm in $C(R)$, if $f \in C(R)$ and

$$\lim_{|x|\to\infty} \rho(x) f(x) = 0 \Rightarrow \lim_{\delta\to 0} \omega(f,\delta) = 0 \quad (13)$$

Preliminaries

In this section, we give some preliminaries, which we shall use in sequal.

$$H_n''(x) - 2xH_n'(x) + 2nH_n(x) = 0 \quad (15)$$

$$H_n'(x) = 2nH_{n-1}(x) \quad (14)$$

$$l_k(x_j) = \begin{cases} 0 & j \ne k \\ \text{for} & k, j = 1,2,3\dots n \\ 1 & j = k \end{cases} \quad (15)$$

$$l_k'(x_j) = \begin{cases} \frac{H_n(x_j)}{H_n'(x_k)(x_j - x_k)} & j \ne k \\ x_k & j = k \end{cases} \quad (16)$$

$$L_k(y_j) = \begin{cases} 0 & j \ne k \\ \text{for} & k, j = 1,2,3\dots n-1 \\ 1 & j = k \end{cases} \quad (17)$$

$$L_k'(y_j) = \begin{cases} \frac{H_n'(y_j)}{H_n''(y_k)(y_j - y_k)} & j \ne k \\ x_k & j = k \end{cases} \quad (18)$$

$k, j = 1,2,3\dots n-1$

For roots of $H_n(x)$ we have

$$x_k^2 \sim \frac{k^2}{n} \qquad k = 1,\ldots,n \quad (19)$$

L.Szili proved

$$\left|x_k^2 - 1\right| = e^{\beta x_k^2} \quad 0 < \beta < 1 \quad (20)$$

$$\left|xe^{-\frac{x^2}{2}} P_n(x)\right| = O(\sqrt{n})\omega(f';\frac{1}{\sqrt{n}}) \quad (21)$$

$$e^{-\frac{x^2}{2}} \left|P_n''(x)\right| = O(\sqrt{n})\omega(f';\frac{1}{\sqrt{n}}) \quad (22)$$

Also we have

$$H_n(x) = O\left\{ n^{-\frac{1}{4}} \sqrt{2^n n!}(1 + \sqrt[3]{|x|}) e^{\frac{x^2}{2}} \right\} x \in R \quad (23)$$

$$\left|H_n'(x_k)\right| \geq c\sqrt{2^{n+1} n!} n^{\frac{1}{4}} e^{\delta \frac{x_k^2}{2}} \quad 0 < \delta < 1,\ k = 1,\ldots,n \quad (24)$$

$$H_n(y_k) \geq c_1 \sqrt{2^{n+1} n!} n^{-\frac{1}{4}} e^{\delta \frac{y_k^2}{2}} \quad 0 < \delta < 1,\ k = 1,\ldots,n-1 \quad (25)$$

$$\sum_{i=1}^{n-1} \frac{H_i(y) H_i(x)}{2^i i!} = \frac{H_n(y) H_{n-1}(x) - H_{n-1}(y) H_n(x)}{2^n (n-1)! (y - x)} \quad (26)$$

At $y = x_k$, we have

$$|l_k(x)| = \frac{O(1) 2^{n+1} n! \sqrt{n} e^{\frac{\nu_1 (x^2 + X_k^2)}{2}}}{H_n'^2(x_k)} \quad \nu_1 > 1 \text{ and } k = 1,\ldots,n \quad (27)$$

$$\left|\int_0^x l_k dt\right| = \frac{O(1) 2^{n+1} n! \log(n) \sqrt{n} e^{\frac{\nu_1 (x^2 + X_k^2)}{2}}}{H_n'^2(x_k)} \quad \nu_1 > 1 \text{ and } k = 1,\ldots,n \quad (28)$$

$$|L_k(x)| = O\left(\frac{2^n n! e^{\frac{\nu_1 (x^2 + X_k^2)}{2}}}{\sqrt{n} H_n^2(y_k)} \right) \quad \nu_1 > 1 \text{ and } k = 1,\ldots,n-1 \quad (29)$$

$$\left|\int_0^x L_k dt\right| = \frac{O(1) 2^{n+1} (n-1)! \log(n) \sqrt{n} e^{\frac{\nu_1 (x^2 + y_k^2)}{2}}}{H_n^2(y_k)} \quad \nu_1 > 1 \text{ and } k = 1,\ldots,n-1 \quad (30)$$

$$\sum_{k=1}^{n} e^{-\varepsilon x_k^2} = O\sqrt{n}) \quad \text{where } \varepsilon > 0 \quad (31)$$

$$\sum_{k=1}^{n} \frac{e^{\delta x_k^2}}{H_n'^2(x_k)} = O\left(\frac{1}{2^{n+1} n!} \right) \quad \text{where } 0 < \delta < 1 \quad (32)$$

$$\frac{2^n \left[\left(\frac{n}{2} \right)! \right]}{n+1} \sim n^{\frac{-1}{2}} \qquad n = 1, 2, 3\ldots.. \quad (33)$$

Proof of Theorem 1

Let

$$R_n(x) = \sum_{n=1}^{n-1} g_k A_k(x) + \sum_{n=1}^{n} g_k^* B_k(x) + \sum_{n=1}^{n} g_k^{**} C_k(x) + DH_n(x) \quad (34)$$

where $A_k(x)$, $B_k(x)$, $C_k(x)$ are polynomials of degree $\leq 3n - 1$, from condition (1.4), one can see that-

$$A_k'(y_j) = \begin{cases} 1 & j \neq k \\ & \text{for} \\ 0 & j = k \end{cases}$$

$$j = 1,\ldots. n - 1;\ A_k(x_j) = 0,\ j = 1,\ldots. n \quad (35)$$

and

$$\left(e^{-\frac{x^2}{2}} A_k(x) \right)''_{x = x_j} = 0 \quad j = 1,\ldots,n$$

$$B_k'(y_j) = 0 \quad j = 1,\ldots,n-1; \qquad B_k(x_j) = \begin{cases} 0 & j \neq k \\ & \text{for} \\ 1 & j = k \end{cases}$$

$$j = 1,\ldots n \quad (36)$$

and

$$\left(e^{-\frac{x^2}{2}} B_k(x) \right)''_{x = x_j} = 0 \quad j = 1,\ldots n$$

$$C_k'(y_j) = 0 \quad j = 1,\ldots,n-1; C_k(x_j) = 0 \quad j = 1,\ldots,n \quad (37)$$

and

$$\left(e^{-\frac{x^2}{2}} C_k(x) \right)''_{x = x_j} = \begin{cases} 0 & j \neq k \\ & \text{for} \qquad j = 1,\ldots n \\ 1 & j = k \end{cases}$$

To determine $C_k(x)$, let

$$C_k(x) = C_1 H_n(x) \int_0^x l_k(t) H_n'(t) dt \quad (38)$$

Where C_1 is a constant. $C_k(x)$ is a polynomial degree $\leq 3n - 1$ and obviously it satisfies conditions (3.4), provided

$$C_1 = \frac{e^{\frac{x_k^2}{2}}}{2H_n'^2(x_k)}$$

To determine $B_k(x)$, let

$$B_k(x) = C_2 H_n'^2(x) l_k(x) - C_3 H_n(x) \int_0^x \{C_4 t l_k'(t) + l_k''(t)\} H_n'(t) dt - C_5 C_k(x$$

Where C_2, C_3, C_4 are arbitrary constants.

Obviously,

$$B_k(x_j)=\begin{cases}0 & j\neq k\\ \text{For} & j=1,\ldots n \text{ and} \qquad B_k^{'}(y_j)=0,\\ 1 & j=k\end{cases}$$

$j=1,\ldots,n-1$

which gives

$$C_2=\frac{1}{H_n^{'2}(x_k)} \tag{39}$$

one can see that

$$\left(e^{-\frac{x^2}{2}}H_n^{'2}(x)l_k(x)\right)^{''}_{x=x_j}=H_n^{'2}(x_j)\,e^{-\frac{x_j^2}{2}}$$

$\{6tl_j^{'}(t)+l_j^{''}(t)\}$ for $j\neq k$

$$\left(e^{-\frac{x^2}{2}}H_n^{'2}(x)l_k(x)\right)^{''}_{x=x_k}=H_n^{'2}(x_k)\,e^{-\frac{x_k^2}{2}}$$

$\{(9x_k^2-4n+3)+6tl_k^{'}(x_k)+l_k^{''}(x_k)\}$ for $j=k$

Obviously for $\left(e^{-\frac{x^2}{2}}B_k(x)\right)^{''}_{x=x_j}=\text{o}$ gives

$$C_3=\frac{1}{2H_n^{'2}(x_k)},\ C_4=6 \text{ and } C_5=e^{-\frac{x_k^2}{2}}(9x_k^2-4n+3)$$

proof of $A_k(x)$ follows on the same lines as $B_k(x)$ so we omit the details.

If n is even, $H_n(0)\neq 0$ so (3.1) at x=0, owing to the last condition of (1.4) gives D = 0

Which completes the proof of theorem.

Estimation of the fundamental polynomials

Lemma: For $x\in(-\infty,+\infty)$ we have

$$\sum_{k=1}^{n}e^{\frac{x_k^2}{2}}\left|C_k(x)\right|=O(n^{\frac{1}{2}})e^{\gamma x^2}\ ,\gamma>1$$

Proof: From (1.2) it follows

We have

$$C_k(x)=\frac{e^{\frac{x_k^2}{2}}}{2H_n^{'^2}(x_k)}H_n(x)\int_0^x l_k(t)H_n^{'}(t)dt$$

$$=\frac{e^{\frac{x_k^2}{2}}}{4H_n^{'}(x_k)}H_n(x)\{l_k(x)(x-x_k)(l_k(x)-l_k(0)\}+\frac{e^{\frac{x_k^2}{2}}}{4H_n^{'}(x_k)}\int_0^x l_k^2(t)dt$$

Thus

$$\sum_{k=1}^{n}e^{\frac{x_k^2}{2}}C_k(x)\leq\sum_{k=1}^{n}\frac{e^{x_k^2}\left|H_n(x)\right|\left|l_k(x)(x-x_k)(l_k(x)-l_k(0)\right|}{4\left|H_n^{'}(x_k)\right|}+\sum_{k=1}^{n}\frac{e^{\frac{x_k^2}{2}}}{4\left|H_n^{'}(x_k)\right|}\left|\int_0^x l_k^2(t)dt\right|$$

owing to (2.7),(2.10),(2.10)(2.19) and (2.22) we get the lemma.

Lemma: For $x\in(-\infty,+\infty)$ we have

$$\sum_{k=1}^{n}e^{\frac{x_k^2}{2}}\left|B_k(x)\right|=O(n^{\frac{1}{2}})e^{\gamma x^2}\ ,\ \gamma>1$$

Proof. It follows from (1.2),(2.1)

$$B_k(x)=\frac{H_n^{'^2}(x)l_k(x)}{H_n^{'^2}(x_k)}-\frac{H_n(x)}{2H_n^{'2}(x_k)}\Bigg[3l_k^{'}(x)(H_n^{'}(x)-H_n^{'}(0)+6nH_n(x)(l_k(x)-l_l(0))-6n\int_0^x l_k(t)H_n^{'}(t)dt-$$

$$\frac{\left|H_n^{'}(x_k)\right|}{2}\left\{l_k^{'}(x)(x-x_k)(l_k^{'}(x)-l_k^{'}(0))-3\int_0^x l_k^{'2}(t)dt+l_k(x)(l_k^{'}(x)-l_k^{'}(0)\right\}\Bigg]+e^{-\frac{x_k^2}{2}}(9x_k^2-4n+3)C_k(x)$$

we have

$$\sum_{k=1}^{n}e^{\frac{x_k^2}{2}}\left|B_k(x)\right|=\sum_{k=1}^{n}\frac{e^{\frac{x_k^2}{2}}H_n^{'^2}(x)\left|l_k(x)\right|}{\left|H_n^{'^2}(x_k)\right|}+\frac{3}{2}\sum_{k=1}^{n}\frac{e^{\frac{x_k^2}{2}}\left|H_n^{'}(x)-H_n^{'}(0)\right|\left|H_n(x)\right|\left|l_k^{'}(x)\right|}{\left|H_n^{'^2}(x_k)\right|}$$

$$+3n\sum_{k=1}^{n}\frac{e^{\frac{x_k^2}{2}}H_n^{'^2}(x)\left|l_k(x)-l_l(0)\right|}{\left|H_n^{'^2}(x_k)\right|}+3n\sum_{k=1}^{n}\frac{e^{\frac{x_k^2}{2}}\left|H_n(x)\right|\left|\int_0^x l_k(t)H_n^{'}(t)dt\right|}{\left|H_n^{'^2}(x_k)\right|}$$

$$-\sum_{k=1}^{n}\frac{e^{\frac{x_k^2}{2}}\left|H_n(x)\right|\left|l_k^{'}(x)(x-x_k)(l_k^{'}(x)-l_k^{'}(0))\right|}{\left|H_n^{'}(x_k)\right|}+\frac{3}{4}\sum_{k=1}^{n}\frac{e^{\frac{x_k^2}{2}}\left|H_n(x)\right|\left|\int_0^x l_k^{'2}(t)dt\right|}{\left|H_n^{'}(x_k)\right|}$$

$$-\frac{1}{4}\sum_{k=1}^{n}\frac{e^{\frac{x_k^2}{2}}\left|H_n(x)\right|\left|l_k(x)\right|\left|l_k^{'}(x)-l_k^{'}(0)\right|}{\left|H_n^{'}(x_k)\right|}+\frac{1}{4}\sum_{k=1}^{n}\frac{e^{\frac{x_k^2}{2}}\left|9x_k^2-4n+3\right|\left|C_k(x)\right|\left|H_n(x)\right|}{\left|H_n^{'}(x_k)\right|}$$

Using (2.7), (2.10),(2.11),(2.12), (2.13), (2.15), (2.18), (2.19),(2.23) and Lemma 4.1,we get the lemma.

Lemma: For $x\in(-\infty,+\infty)$ we have

$$\sum_{k=1}^{n}e^{\frac{x_k^2}{2}}\left|A_k(x)\right|=O(n^{\frac{1}{2}})e^{\gamma x^2}$$

Proof. By (1.3) it follows

$$A_k(x)=$$

$$\frac{H_n(x)}{4y_kH_n(y_k)}\left\{L_k^2(x)-L_k^2(0)\right\}+\frac{L_k^2(x)H_n(x)}{2y_kH_n(y_k)}+B_k(x)$$

Thus

$$\sum_{k=1}^{n}e^{\frac{y_k^2}{2}}\left|A_k(x)\right|\leq\frac{1}{4}\sum_{k=1}^{n-1}\frac{e^{\frac{y_k^2}{2}}\left|H_n(x)\right|\left|L_k^2(x)-L_k^2(0)\right|}{y_k\left|H_n(y_k)\right|}+\frac{1}{2}\sum_{k=1}^{n-1}\frac{\left|H_n(x)\right|\left|L_k^2(x)\right|}{y_k\left|H_n(y_k)\right|}+\sum_{k=1}^{n-1}e^{\frac{y_k^2}{2}}\left|B_k(x)\right|$$

Using (2.7), (2.10), (2.18), (2.19) and Lemma 4.2 we get required Lemma

Theorem **(**G. Freud[3]**).** Let $f:R\to R$ .be continuously differentiable. Further let

$Lim x^{2r} \rho(x) f(x) = 0,\ r = 0,1,2....$ and

$Lim \rho(x) f^{'}(x) = 0$

Then there exist polynomials $P_n(x)$ of degree $\leq n$ such that

$$\rho(x)\left|f(x) - P_n(x)\right| = O\left(\frac{1}{\sqrt{n}}\right)\omega\left(f^{'};\frac{1}{\sqrt{n}}\right),\ x \in R \quad (40)$$

and

$$\rho(x)\left|f^{'}(x) - P_n^{'}(x)\right| = O(1)\omega\left(f^{'};\frac{1}{\sqrt{n}}\right),\ x \in R \quad (41)$$

Szili establish the following

$$\rho(x)\left|P_n^{(r)}(x) = O(1)\right| \quad (42)$$

where ω stands for modulus of continuity defined by (1.12) and $\rho(x)$ is the weight function.

Proof of Theorem 2

Since $R_n(x)$ given by (1.5) is exact for all $P_n(x)$ of degree $\leq 3n-1$, we have

$$P_n(x) = \sum_{n=1}^{n-1} P_n^{'}(y_k)A_k(x) + \sum_{n=1}^{n} P_n(x_k)B_k(x) + \sum_{n=1}^{n}\left(e^{\frac{x^2}{2}}P_n(x)\right)^{''} C_k(x) + d_n H_n(x)H_n^{'}(x) \quad (43)$$

Where

$$d_n = \frac{1}{\left(H_n^{'}(0)\right)^2}\left[P_n^{'}(0) - \sum_{n=1}^{n-1}\frac{P_n(y_k)L_k^2(0)H_n^{'}(0)}{y_k H_n(y_k)}\right] \quad (44)$$

One can easily see that

$$e^{-\nu x^2}\left|R_n(x) - f(x)\right| = O(e^{-\nu x^2})\left\{\left|P_n(x) - f(x)\right| + \sum_{n=1}^{n-1}\left|f^{'}(y_k) - P_n^{'}(y_k)\right|\left|A_k(x)\right|\right.$$
$$\left. + \sum_{k=1}^{n}\left|f(x_k) - P_n(y_k)\right|\left|B_k(x)\right| + \sum_{k=1}^{n}\left\{\delta_k - \left(e^{\frac{x^2}{2}}P_n(x)\right)^{''}\right\}\left|C_k(x)\right| + \left|d_n H_n(x)H_n^{'}(x)\right|\right\}$$

Thus from (5.1),(5.2),(5.3),(1.10), (2.1). (2.2).(2.8), (2.9),(2.10), (2.19), (2.11),(2.20),Lemma 4.1, Lemma 4.2,Lemma 4.3 and (6.2) completes the proof.

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