# DIMENSIONAL SYNTHESIS OF EIGHT-BAR MECHANISM BASED ON PATH GENERATION 

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#### Abstract

This paper presents dimensional synthesis procedure for an eight-bar, single degree of freedom, 10R mechanism that traces a specified trajectory. The mechanism is dimensionally synthesized to solve the problem of generating a path that comprises of a given number of precision points. During the synthesis, various standard dyad/triad and analytical loop closure equations are formed and generated for different displaced positions of the mechanism. These equations are written using complex numbers which presents a perfect tool for the purpose of modelling planar kinematic linkages. The solution of these equations is obtained with the help of a MATLAB code. The dimensional synthesis of the eight-bar mechanism is demonstrated and verified by a numerical example. The final dimensions and orientations of different links of the mechanism has been graphically plotted and presented in results.


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## INTRODUCTION

Inventing useful mechanism for achieving desired performance is always needed to augment the requirements of automation industry. In the industry, sophisticated industrial machining processes and devices require movement of working member or link to be prescribed along a dedicated path in order to achieve desired accuracy in their operations. Dimensional synthesis of linkage is an important step to design such machines that carry out machining processes with desired accuracy. It involves determination of principal dimensions of the mechanism that may have any number of links or the mechanism may be a slider crank linkage. Depending upon the requirement, the kinematic task may be generation of prescribed function, path, motion or guidance of rigid body etc. The determination of principal dimensions involves finding the lengths and angular orientations of all links, position of pivots and displacement between extreme positions of slider mechanism. Both graphical and analytical techniques have been applied by various researchers to carry out dimensional syntheses of mechanisms. The graphical methods have limitation of drawing accuracy and were prevalent till starting of $20^{\text {th }}$ century second-half $[1,2]$. On the other hand, analytical methods when used along with high-end programming software are simpler, easy to apply and provide benefits of improved accuracy. The dimensional syntheses of mechanisms have been carried out by various researchers using graphical and analytical methods. A five-bar mechanism was dimensionally synthesized using graphical methods by Rose and Rawat [1-2].

[^0]An adjustable four bar mechanism was suggested for synthesis using five-bar loop closure equations by Naik and Amarnath [3]. The variable topology planar five-bar mechanisms were synthesized by Balli and Chand [4-5] using complex number mathematics. Their synthesis work was meant for motion between extreme positions and later, it was extended to the five-bar mechanism with two binary offset tracing points links [6-7]. Using complex numbers, the variable topology sevenbar mechanisms were synthesized by various researchers [811]. They carried out these syntheses for motion between extreme positions based on various kinematic tasks viz., function generation, motion generation and path generation. Furthermore, the variable topology mechanisms of five-bar slider and seven-bar slider were jointly synthesized by Daivagna and Balli [12-14]. They projected the synthesis for finitely separated positions

The six-bar 1-DOF mechanisms were synthesized by different researchers [15-17]. Their synthesis work was confined upto a certain number of precision points. The dimensional synthesis of six-bar mechanism based on path generation was carried out [15]. This work was based on 8 precision points and later, it was extended to 12 and 15 precision points [16-17]. Recently, the dimensional synthesis of six-bar Watt mechanisms was carried out for 20 and 24 precision points [18-19]. Also, fourbar and different six-bar mechanisms were optimally synthesized using various optimization techniques [20-22], but the working members of these mechanisms do not exactly generate the prescribed path because of inaccuracy in coupler tracing point following. In fact, there exists a difference between the desired path prescribed by the designer and the actual path followed by working link (i.e. coupler) of mechanism i.e. there exist a variation between the two paths.

To reduce this variation, one possible solution is to increase the number of links. But, this introduces complexity in determining the solution of the synthesis problem as the number of analytical loop closure equations is also increased. By incorporating the high-end programming software, the exact solution to such problems can be easily obtained. Therefore, in present work, the dimensional synthesis of an eight-bar mechanism is carried out that traces a trajectory defined by given number of precision points. The analytical loop closure equations have been generated for different dyads and triads using complex number mathematics. The effectiveness of the method is demonstrated on a numerical example which is solved for eight-bar mechanism whose coupler tracing point traces a trajectory defined by twelve precision points. Finally, the solution is verified using SAM software.

## Configuration of Eight-Bar Mechanism

An eight-bar single degree of freedom mechanism has 8 links and 10 joints with all revolute pairs. In the present work, the given Eight-bar mechanism is characterised by 4 binary and 4 ternary links. The fixed link of the mechanism is binary link and it directly forms revolute pairs at each end with one of the end of two different ternary links. The configuration of eightbar mechanism is shown in Fig. 1. The link 1 is binary link which is fixed at pivots $\mathrm{O}_{1}$ and $\mathrm{O}_{2}$. The link 2 is a ternary crank link and rotates about pivot point $\mathrm{O}_{1}$. The input motion is supplied to the mechanism through ternary crank link 2 . The link 3 is a ternary link, which is connected with three ternary links at joints A, D and C. The link 4 is a ternary link, which is connected with two binary links and one ternary link at joints G, E and B respectively. The link 5 is a binary link with an offset at point F which is the tracing point of the mechanism for which it is dimensionally synthesized. There is another ternary link 6 , which is connected with three binary links at joints $\mathrm{O}_{2}, \mathrm{H}$ and I. The link 7 is a binary link, which is connected with two ternary links i.e. link 3 and link 6 at joints C and I respectively. The link 8 is also a binary link, which is connected with two ternary links i.e. link 4 and link 6 at joints G and H respectively.


Fig 1 Configuration of Eight-Bar Mechanism


Fig 2 Eight-Bar Mechanism displaced from home to prime position by $\delta_{\mathrm{j}}$

## Generation of Loop Closure Equations

Consider the initial arrangement of the given eight-bar mechanism expressed by joint positions $\mathrm{O}_{1} \mathrm{~A}_{0} \mathrm{~B}_{0} \mathrm{C}_{0} \mathrm{D}_{0} \mathrm{E}_{0} \mathrm{~F}_{0} \mathrm{G}_{0} \mathrm{H}_{0} \mathrm{I}_{0} \mathrm{O}_{2}$ (Refer Fig. 2). When the ternary crank link $\mathrm{O}_{1} \mathrm{~A}_{0} \mathrm{~B}_{0}$ rotates through angle $\theta$ and reaches position $\mathrm{O}_{1} \mathrm{~A}_{\mathrm{j}} \mathrm{B}_{\mathrm{j}}$, the joint positions of remaining links are also shifted and prime position of the linkage is expressed by $\mathrm{O}_{1} \mathrm{~A}_{\mathrm{j}} \mathrm{B}_{\mathrm{j}} \mathrm{C}_{\mathrm{j}} \mathrm{D}_{\mathrm{j}} \mathrm{E}_{\mathrm{j}} \mathrm{F}_{\mathrm{j}} \mathrm{G}_{\mathrm{j}} \mathrm{H}_{\mathrm{j}} \mathrm{I}_{\mathrm{j}} \mathrm{O}_{2}$ as shown in Fig. 2. The displacement of tracing point from position $\mathrm{F}_{0}$ to $\mathrm{F}_{\mathrm{j}}$ is expressed by $\delta_{\mathrm{j}}$. The derivation of loop closure equations for a given number of precision points is explained below:
Writing the loop closure equation [23] for independent vector loop $\mathrm{O}_{1} \mathrm{~B}_{\mathrm{j}} \mathrm{E}_{\mathrm{j}} \mathrm{F}_{\mathrm{j}} \mathrm{F}_{0} \mathrm{E}_{0} \mathrm{~B}_{0} \mathrm{O}_{1}$ (Refer Fig. 2)

$$
\begin{align*}
& Z_{2} e^{i \theta_{j}}+Z_{7} e^{i \beta_{j}}+Z_{12} e^{i \gamma_{j}}-\delta_{j}-Z_{12}-Z_{7}-Z_{2}=0 \\
& Z_{2}\left(e^{i \theta_{j}}-1\right)+Z_{7}\left(e^{i \beta_{j}}-1\right)+Z_{12}\left(e^{i \gamma_{j}}-1\right)=\delta_{j} \tag{1}
\end{align*}
$$

Writing the loop closure equation [23] for independent vector loop $\mathrm{O}_{1} \mathrm{~A}_{\mathrm{j}} \mathrm{D}_{\mathrm{j}} \mathrm{F}_{\mathrm{j}} \mathrm{F}_{0} \mathrm{D}_{0} \mathrm{~A}_{0} \mathrm{O}_{1}$ (Refer Fig. 2)
$Z_{1} e^{i \theta_{j}}+Z_{5} e^{i \alpha_{j}}+Z_{10} e^{i \gamma_{j}}-\delta_{j}-Z_{10}-Z_{5}-Z_{1}=0$
$Z_{1}\left(e^{i \theta_{j}}-1\right)+Z_{5}\left(e^{i \alpha_{j}}-1\right)+Z_{10}\left(e^{i \gamma_{j}}-1\right)=\delta_{j}$
Writing the loop closure equation [23] for independent vector loop $\mathrm{O}_{2} \mathrm{I}_{\mathrm{j}} \mathrm{C}_{\mathrm{j}} \mathrm{D}_{\mathrm{j}} \mathrm{F}_{\mathrm{j}} \mathrm{F}_{0} \mathrm{D}_{0} \mathrm{C}_{0} \mathrm{I}_{0} \mathrm{O}_{2}$ (Refer Fig. 2)
$Z_{15} e^{i \varphi_{j}}+Z_{10} e^{i \lambda_{j}}+Z_{6} e^{i \alpha_{j}}+Z_{10} e^{i Y_{j}}-\delta_{j}-Z_{10}-Z_{6}-Z_{16}-Z_{15}=0$
$Z_{15}\left(e^{i \varphi_{j}}-1\right)+Z_{16}\left(e^{i \lambda_{j}}-1\right)+Z_{6}\left(e^{i \alpha_{j}}-1\right)+Z_{10}\left(e^{i \gamma_{j}}-1\right)=\delta_{j}$

Writing the loop closure equation [23] for independent vector loop $\mathrm{O}_{2} \mathrm{H}_{\mathrm{j}} \mathrm{G}_{\mathrm{j}} \mathrm{E}_{\mathrm{j}} \mathrm{F}_{\mathrm{j}} \mathrm{F}_{0} \mathrm{E}_{0} \mathrm{G}_{0} \mathrm{H}_{0} \mathrm{O}_{2}$ (Refer Fig. 2)
$Z_{13} e^{i \varphi_{j}}+Z_{17} e^{i \mu_{j}}+Z_{9} e^{i \beta_{j}}+Z_{12} e^{i \gamma_{j}}-\delta_{j}-Z_{12}-Z_{9}-Z_{17}-Z_{13}=0$
$Z_{13}\left(e^{i \varphi_{j}}-1\right)+Z_{17}\left(e^{i \mu_{j}}-1\right)+Z_{9}\left(e^{i \beta_{j}}-1\right)+Z_{12}\left(e^{i \gamma_{j}}-1\right)=\delta_{j}$

In eq ${ }^{\text {ns }}(1)$ to (4), value of $j$ varies from 1 to 6 for six precision points comprising the required trajectory.
Considering closed loop $\mathrm{O}_{1} \mathrm{~A}_{0} \mathrm{~B}_{0} \mathrm{O}_{1}$, we get unknown vector $Z_{3}$ (Refer Fig. 2)
$Z_{3}=Z_{2}-Z_{1}$

Considering closed loop $\mathrm{D}_{0} \mathrm{E}_{0} \mathrm{~F}_{0} \mathrm{D}_{0}$, we get unknown vector $Z_{11}$ (Refer Fig. 2)
$Z_{11}=Z_{10}-Z_{12}$

Considering closed loop $\mathrm{B}_{0} \mathrm{G}_{0} \mathrm{E}_{0} \mathrm{~B}_{0}$, we get unknown vector $Z_{8}$ (Refer Fig. 2)
$Z_{8}=Z_{7}-Z_{9}$

Considering closed loop $\mathrm{A}_{0} \mathrm{C}_{0} \mathrm{D}_{0} \mathrm{~A}_{0}$, we get unknown vector $Z_{4}$ (Refer Fig. 2)
$Z_{4}=Z_{5}-Z_{6}$
Considering closed loop $\mathrm{O}_{2} \mathrm{H}_{0} \mathrm{I}_{0} \mathrm{O}_{2}$, we get unknown vector $Z_{14}$ (Refer Fig. 2)
$Z_{14}=Z_{15}-Z_{13}$
Considering closed loop $\mathrm{O}_{1} \mathrm{O}_{2} \mathrm{I}_{0} \mathrm{C}_{0} \mathrm{~A}_{0} \mathrm{O}_{1}$, we get unknown vector $Z_{18}$ (Refer Fig. 2)
$Z_{18}=Z_{1}+Z_{4}-Z_{15}-Z_{16}$

## Numerical Problem Based On Synthesis of Eight-Bar Mechanism

Problem Statement: It is required to synthesize a Eight-bar mechanism which transmits motion along a path prescribed by six precision points (Refer Fig. 3) $\mathrm{P}_{1}(0.599,0.593), \mathrm{P}_{2}(0.330$, $0.554), \mathrm{P}_{3}(0.234,0.349), \mathrm{P}_{4}(0.330,0.261), \mathrm{P}_{5}(0.545,0.326)$ and $\mathrm{P}_{6}(0.741,0.418)$.
Prescribed Parameters: The prescribed parameters are displacement of each point from initial position i.e. $\delta_{j}=P_{j}-P_{1}$ at equal intervals of $\theta_{j}\{0,2 \pi\}$ (where $\mathrm{j}=1,2, \ldots 6$ ).

Assumed Parameters: The parameters assumed freely are $\alpha_{j}$, $\beta_{j}, \gamma_{j}, \lambda_{j}$ and $\mu_{j}$. The range of these parameters are $\alpha_{j}\{-2 \pi / 30$, $\pi / 5\}, \beta_{\mathrm{j}}\{-\pi / 180, \pi / 4\}, \gamma_{\mathrm{j}}\{-5 \pi / 36, \pi / 20\}, \phi_{\mathrm{j}}\{-\pi / 30, \pi / 3\}, \lambda_{\mathrm{j}}\{-$ $\pi / 30,11 \pi / 50\}$ and $\mu_{\mathrm{j}}\{-\pi / 20, \pi / 5\}$ [24].
Design Parameters: The MATLAB code is developed to solve for the design vectors $Z_{1}, Z_{2}, Z_{3}, Z_{4}, Z_{5}, Z_{6}, Z_{7}, Z_{8}, Z_{9}, Z_{10}, Z_{11}$, $Z_{12}, Z_{13}, Z_{14}, Z_{15}, Z_{16}, Z_{17}$ and $Z_{18}$.

## Solution of Loop Closure Equations

The solution of equations (1) to (10) is difficult to find manually for large number of precision points. Also, the number of equations is more than the number of unknowns, so a code is developed in MATLAB to solve these equations. To obtain solution, MATLAB algorithm consists of following steps:
Step1. Read the value of given number of precision points coordinates.
Step2. Calculate displacement $\left(\delta_{j}\right)$ for each point by subtracting its coordinates from initial point.


Fig 3 Mobility of Eight-Bar Mechanism from initial position to 6 displaced positions

Step3. Read the values of all free parameters and assumed parameters.
Step4. Calculate values $e^{i \theta_{j}}, e^{i \alpha_{j}}, e^{i \beta_{j}}, e^{i \gamma_{j}}, e^{i \varphi_{j}}, e^{i \lambda_{j}}$ and $e^{i \mu_{j}}$ for $\mathrm{j}=1,2, \ldots . . .6$.
Step5. Calculate design vectors $Z_{2}, Z_{7}$ and $Z_{12}$ using eq. (1).
Step6. Calculate design vectors $Z_{1}, Z_{5}$ and $Z_{10}$ using eq. (2).

Step 7. Calculate design vectors $Z_{15}, Z_{16}$ and $Z_{6}$ using eq. (3).
Step8. Calculate design vectors $Z_{13}, Z_{17}$ and $Z_{9}$ using eq. (4).
Step9. Calculate design vector $Z_{3}$ using eq. (5).
Step10. Calculate design vector $Z_{11}$ using eq. (6).
Step11. Calculate design vector $Z_{8}$ using eq. (7).
Step12. Calculate design vector $Z_{4}$ using eq. (8).
Step13. Calculate design vector $Z_{14}$ using eq. (9).
Step14. Calculate design vector $Z_{18}$ using eq. (10).

## RESULTS AND DISCUSSION

The dimensions and orientations of various link lengths obtained by solving loop closure equations using MATLAB code are:
$\mathrm{Z}_{1}=0.238 \mathrm{~m} \angle 61.94^{\circ} ; \mathrm{Z}_{2}=0.266 \mathrm{~m} \angle 79.58^{\circ} ; \mathrm{Z}_{3}=0.082 \mathrm{~m}$ $\angle 141.35^{\circ} ; \mathrm{Z}_{4}=0.232 \mathrm{~m} \angle 345.49^{\circ} ; \mathrm{Z}_{5}=0.297 \mathrm{~m} \angle 9.40^{\circ} ; \mathrm{Z}_{6}=$ $0.126 \mathrm{~m} \angle 57.47^{\circ} ; \mathrm{Z}_{7}=0.370 \angle 26.53^{\circ} ; \mathrm{Z}_{8}=0.781 \mathrm{~m} \angle 31.86^{\circ}$; $\mathrm{Z}_{9}=0.415 \mathrm{~m} \angle 36.70^{\circ} ; \mathrm{Z}_{10}=0.305 \mathrm{~m} \angle 45.49^{\circ} ; \mathrm{Z}_{11}=0.170 \mathrm{~m}$ $\angle 98.95^{\circ} ; \mathrm{Z}_{12}=0.245 \mathrm{~m} \angle 11.63^{\circ} ; \mathrm{Z}_{13}=0.304 \mathrm{~m} \angle 83.82^{\circ} ; \mathrm{Z}_{14}=$ $0.185 \mathrm{~m} \angle 118.37^{\circ} ; Z_{15}=0.469 \mathrm{~m} \angle 96.77^{\circ} ; Z_{16}=0.456 \mathrm{~m}$ $\angle 223.40^{\circ} ; \mathrm{Z}_{17}=0.374 \mathrm{~m} \angle 96.89^{\circ} ; \mathrm{Z}_{18}=0.724 \mathrm{~m} \angle 0.08^{\circ}$.

The final dimensions of each link of eight-bar mechanism based on path generation are graphically shown in Fig. 4.


Fig 4 Graphical representation of the final dimensions of each link of Eight-Bar Mechanism based on path generation

## CONCLUSIONS

The present work suggests dimensional synthesis of an eightbar mechanism that transmits motion for a specified path comprised by given number of precision points. This synthesis work augments the requirements of automation industry to carry out sophisticated industrial machining processes effectively and efficiently. It will facilitate in the invention of useful mechanisms in which the movement of working member or link is along a dedicated path to achieve desired accuracy in different industrial operations. The method offers reduced solution space with increased accuracy in results and signifies the effectiveness of the proposed method. The solution obtained is consistent with kinematic tasks of path generation and is further extendable for other kinematic tasks of motion generation and function generation.

## Nomenclature

$Z_{i} \quad:$ Length of each side of all of links (i=1,2,...18)
$\delta_{j} \quad$ : Displacement of tracing point from position $\mathrm{F}_{0}$ to $\mathrm{F}_{\mathrm{j}}$ ( $\mathrm{j}=1,2, \ldots 6$ )
$\theta_{j} \quad:$ Angle through which ternary crank link $\mathrm{O}_{1} \mathrm{~A}_{0} \mathrm{~B}_{0}$ rotates and reaches at position $\mathrm{O}_{1} \mathrm{~A}_{\mathrm{j}} \mathrm{B}_{\mathrm{j}}$
$\alpha_{j} \quad:$ Angle through which ternary link $\mathrm{A}_{0} \mathrm{C}_{0} \mathrm{D}_{0}$ rotates and reaches at position $\mathrm{A}_{\mathrm{j}} \mathrm{C}_{\mathrm{j}} \mathrm{D}_{\mathrm{j}}$
$\beta_{j} \quad$ : Angle through which ternary link $\mathrm{B}_{0} \mathrm{E}_{0} \mathrm{G}_{0}$ rotates and reaches at position $\mathrm{B}_{\mathrm{j}} \mathrm{E}_{\mathrm{j}} \mathrm{G}_{\mathrm{j}}$
$\gamma_{j} \quad:$ Angle through which ternary link $\mathrm{D}_{0} \mathrm{E}_{0} \mathrm{~F}_{0}$ rotates and reaches at position $\mathrm{D}_{\mathrm{j}} \mathrm{E}_{\mathrm{j}} \mathrm{F}_{\mathrm{j}}$
$\phi_{j} \quad:$ Angle through which ternary link $\mathrm{O}_{2} \mathrm{H}_{0} \mathrm{I}_{0}$ rotates and reaches at position $\mathrm{O}_{2} \mathrm{H}_{\mathrm{j}} \mathrm{I}_{\mathrm{j}}$
$\lambda_{j} \quad:$ Angle through which binary link $\mathrm{C}_{0} \mathrm{I}_{0}$ rotates and reaches at position $\mathrm{C}_{\mathrm{j}} \mathrm{I}_{\mathrm{j}}$
$\mu_{j} \quad$ : Angle through which binary link $\mathrm{G}_{0} \mathrm{H}_{0}$ rotates and reaches at position $\mathrm{G}_{\mathrm{j}} \mathrm{H}_{\mathrm{j}}$

## Appendix

The loop closure equation (1) for eight-bar mechanism is $Z_{2}\left(e^{i \theta_{j}}-1\right)+Z_{7}\left(e^{i \beta_{j}}-1\right)+Z_{12}\left(e^{i \gamma_{j}}-1\right)=\delta_{j}$ for six precision points, say ( $\mathrm{j}=1,2 \ldots \ldots \ldots . .6$ );
The above equation can be formulated in the form as

$$
\begin{align*}
& Z_{2}\left(e^{i \theta_{1}}-1\right)+Z_{7}\left(e^{i \beta_{1}}-1\right)+Z_{12}\left(e^{i \gamma_{1}}-1\right)=\delta_{1} \\
& Z_{2}\left(e^{i \theta_{2}}-1\right)+Z_{7}\left(e^{i \beta_{2}}-1\right)+Z_{12}\left(e^{i \gamma_{2}}-1\right)=\delta_{2}  \tag{11}\\
& Z_{2}\left(e^{i \theta_{3}}-1\right)+Z_{7}\left(e^{i \beta_{3}}-1\right)+Z_{12}\left(e^{i \gamma_{3}}-1\right)=\delta_{3}  \tag{12}\\
& Z_{2}\left(e^{i \theta_{4}}-1\right)+Z_{7}\left(e^{i \beta_{4}}-1\right)+Z_{12}\left(e^{i \gamma_{4}}-1\right)=\delta_{4}  \tag{13}\\
& Z_{2}\left(e^{i \theta_{5}}-1\right)+Z_{7}\left(e^{i \beta_{5}}-1\right)+Z_{12}\left(e^{i \gamma_{5}}-1\right)=\delta_{5}  \tag{14}\\
& Z_{2}\left(e^{i \theta_{6}}-1\right)+Z_{7}\left(e^{i \beta_{6}}-1\right)+Z_{12}\left(e^{i \gamma_{6}}-1\right)=\delta_{6} \tag{15}
\end{align*}
$$

The loop closure equation (2) for eight-bar mechanism is

$$
Z_{1}\left(e^{i \theta_{j}}-1\right)+Z_{5}\left(e^{i \alpha_{j}}-1\right)+Z_{10}\left(e^{i \gamma_{j}}-1\right)=\delta_{j}
$$

for six precision points, say $(j=1,2 \ldots \ldots \ldots . .6)$;
The above equation can be formulated in the form as

$$
\begin{align*}
& Z_{1}\left(e^{i \theta_{1}}-1\right)+Z_{5}\left(e^{i \alpha_{1}}-1\right)+Z_{10}\left(e^{i \gamma_{1}}-1\right)=\delta_{1} \\
& Z_{1}\left(e^{i \theta_{2}}-1\right)+Z_{5}\left(e^{i \alpha_{2}}-1\right)+Z_{10}\left(e^{i \gamma_{2}}-1\right)=\delta_{2}  \tag{17}\\
& Z_{1}\left(e^{i \theta_{3}}-1\right)+Z_{5}\left(e^{i \alpha_{3}}-1\right)+Z_{10}\left(e^{i \gamma_{3}}-1\right)=\delta_{3}  \tag{18}\\
& Z_{1}\left(e^{i \theta_{4}}-1\right)+Z_{5}\left(e^{i \alpha_{4}}-1\right)+Z_{10}\left(e^{i \gamma_{4}}-1\right)=\delta_{4}  \tag{19}\\
& Z_{1}\left(e^{i \theta_{5}}-1\right)+Z_{5}\left(e^{i \alpha_{5}}-1\right)+Z_{10}\left(e^{i \gamma_{5}}-1\right)=\delta_{5} \tag{20}
\end{align*}
$$

$$
\begin{equation*}
Z_{1}\left(e^{i \theta_{6}}-1\right)+Z_{5}\left(e^{i \alpha_{6}}-1\right)+Z_{10}\left(e^{i \gamma_{6}}-1\right)=\delta_{6} \tag{21}
\end{equation*}
$$

The loop closure equation (3) for eight-bar mechanism is
$Z_{15}\left(e^{i \varphi_{j}}-1\right)+Z_{16}\left(e^{i \lambda_{j}}-1\right)+Z_{6}\left(e^{i \alpha_{j}}-1\right)+Z_{10}\left(e^{i \gamma_{j}}-1\right)=\delta_{j}$ for six precision points, say ( $\mathrm{j}=1,2 \ldots \ldots \ldots . .6$ );
The above equation can be formulated in the form as

$$
\begin{align*}
& Z_{15}\left(e^{i \varphi_{1}}-1\right)+Z_{16}\left(e^{i \lambda_{1}}-1\right)+Z_{6}\left(e^{i \alpha_{1}}-1\right)+Z_{10}\left(e^{i \gamma_{1}}-1\right)=\delta_{1} \\
& Z_{15}\left(e^{i \varphi_{2}}-1\right)+Z_{16}\left(e^{i \lambda_{2}}-1\right)+Z_{6}\left(e^{i \alpha_{2}}-1\right)+Z_{10}\left(e^{i \gamma_{2}}-1\right)=\delta_{2} \tag{23}
\end{align*}
$$

$$
\begin{equation*}
Z_{15}\left(e^{i \varphi_{3}}-1\right)+Z_{16}\left(e^{i \lambda_{3}}-1\right)+Z_{6}\left(e^{i \alpha_{3}}-1\right)+Z_{10}\left(e^{i \gamma_{3}}-1\right)=\delta_{3} \tag{24}
\end{equation*}
$$

$$
\begin{equation*}
Z_{15}\left(e^{i \varphi_{4}}-1\right)+Z_{16}\left(e^{i \lambda_{4}}-1\right)+Z_{6}\left(e^{i \alpha_{4}}-1\right)+Z_{10}\left(e^{i \gamma_{4}}-1\right)=\delta_{4} \tag{25}
\end{equation*}
$$

$Z_{15}\left(e^{i \varphi_{5}}-1\right)+Z_{16}\left(e^{i \lambda_{5}}-1\right)+Z_{6}\left(e^{i \alpha_{5}}-1\right)+Z_{10}\left(e^{i \gamma_{5}}-1\right)=\delta_{5}$

$$
\begin{equation*}
Z_{15}\left(e^{i \varphi_{6}}-1\right)+Z_{16}\left(e^{i \lambda_{6}}-1\right)+Z_{6}\left(e^{i \alpha_{6}}-1\right)+Z_{10}\left(e^{i \gamma_{6}}-1\right)=\delta_{6} \tag{27}
\end{equation*}
$$

The loop closure equation (3) for eight-bar mechanism is $Z_{13}\left(e^{i \varphi_{j}}-1\right)+Z_{17}\left(e^{i \mu_{j}}-1\right)+Z_{9}\left(e^{i \beta_{j}}-1\right)+Z_{12}\left(e^{i \gamma_{j}}-1\right)=\delta_{j}$
for six precision points, say ( $\mathrm{j}=1,2 \ldots \ldots \ldots . .6$ );
The above equation can be formulated in the form as

$$
\begin{align*}
& Z_{13}\left(e^{i \varphi_{1}}-1\right)+Z_{17}\left(e^{i \mu_{1}}-1\right)+Z_{9}\left(e^{i \beta_{1}}-1\right)+Z_{12}\left(e^{i \gamma_{1}}-1\right)=\delta_{1} \\
& Z_{13}\left(e^{i \varphi_{2}}-1\right)+Z_{17}\left(e^{i \mu_{2}}-1\right)+Z_{9}\left(e^{i \beta_{2}}-1\right)+Z_{12}\left(e^{i \gamma_{2}}-1\right)=\delta_{2}  \tag{299}\\
& Z_{13}\left(e^{i \varphi_{3}}-1\right)+Z_{17}\left(e^{i \mu_{3}}-1\right)+Z_{9}\left(e^{i \beta_{3}}-1\right)+Z_{12}\left(e^{i \gamma_{3}}-1\right)=\delta_{3}  \tag{30}\\
& Z_{13}\left(e^{i \varphi_{4}}-1\right)+Z_{17}\left(e^{i \mu_{4}}-1\right)+Z_{9}\left(e^{i \beta_{4}}-1\right)+Z_{12}\left(e^{i \gamma_{4}}-1\right)=\delta_{4}  \tag{31}\\
& Z_{13}\left(e^{i \varphi_{5}}-1\right)+Z_{17}\left(e^{i \mu_{5}}-1\right)+Z_{9}\left(e^{i \beta_{5}}-1\right)+Z_{12}\left(e^{i \gamma_{5}}-1\right)=\delta_{5}  \tag{32}\\
& Z_{13}\left(e^{i \varphi_{6}}-1\right)+Z_{17}\left(e^{i \mu_{6}}-1\right)+Z_{9}\left(e^{i \beta_{6}}-1\right)+Z_{12}\left(e^{i \gamma_{6}}-1\right)=\delta_{6} \tag{33}
\end{align*}
$$

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