CRITICAL GRAPHS OF S-VALUED GRAPHS

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A B S T R A C T

In [4], the authors introduced the notion of semi ring valued graphs. In [3], the authors introduced the notion of regularity on S-Valued graphs. In [5], we have introduced the notion of coloring on S-valued graphs. In [6], we have introduced the notion of K-coloring on S-valued graphs. In this paper, we study the critical graphs of S-valued graphs.

INTRODUCTION

The problem of coloring of a graph is equivalent to the problem of partitioning the vertex set into subsets, where each subset consist of vertices of the same color. This problem of colorings finds its application in storage of chemicals, or matching problems, scheduling problems. The problem of coloring in crisp graph is dealt in [2] by Jenson. In [4] the authors introduced the notion of semi ring valued graphs. In [5] we have introduced the notion of colorings on S-valued graphs. In [6] we have introduced the notion of K-coloring on S-valued graphs. In [7] we have introduced the notion of chromatic number of some S-valued graphs. In this paper we study the concept critical graphs in some S-valued graphs.

Preliminaries

In this section, we recall some basic definitions that are required for our work.

Definition [1]: Let (S, +,.) be a semiring. ‘≤’ is said to be a canonical preorder if for a, b ∈ S, a ≤ b if and if there exists c ∈ S such that a + c = b

Definition [2]: A k – vertex colorings of a graph G is an assignment of k – colors to the vertices of G such that no two adjacent vertices receive the same color.

Definition [2]: A graph G that required k – different colors for its colorings and not less number of colors is called a k – chromatic graph and the number k is called the chromatic number of G, denoted by χ(G). That is χ(G) = k.

Definition: A graph G is called critical if χ(H) < χ(G) for every proper subgraph H of G.

Definition: A graph G is called k- critical if χ(G)=k and for each v∈V(G), χ(G − v) < χ(G)

Definition [3]: Let G = (V, E) be a given graph with V, E ≠ Ø.
For any semi ring (S, +,.) a S-valued graph (or S-valued graph) G^S is defined to be the graph G^S = (V, E, σ, ψ) where σ : V → S and ψ : E → S is defined to be
ψ(x, y) = \begin{cases} \min\{σ(x), σ(y)\} & \text{if } σ(x) ≤ σ(y) \text{ or } σ(y) ≤ σ(x) \\ 0 & \text{otherwise} \end{cases}
for every unordered pair (x, y) of E ⊆ V × V. We call σ, a S-vertex set and ψ a S-edge set of the S-valued graph G^S.

Definition [5]: Consider the S-valued graph G^S = (V, E, σ, ψ). A colouring of G^S is given by a function f: V → S×C such that for all v∈V, f(v) = (σ(v), c(v)), c(v) ∈ C.

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**Definition [5]:** A coloring $f: V \times V \rightarrow S \times C$ is said to be proper weight-uni coloring, if $\forall v \in V$ and $c(v) \in C$ is the same, but $\sigma(v) \in S$ differ for adjacent vertices.

**Definition [5]:** Consider a $S$-valued graph $G^S$. A coloring $f$ on $G^S$ is said to be equi-weight (or vertex regular) proper coloring if for all $v \in V$, $\sigma(v)$ have equal value in $S$ and $c(v) \in C$ differ for adjacent vertices.

**Definition [5]:** Consider a $S$-valued graph $G^S$. A coloring $f$ on $G^S$ is said to be total proper coloring if for all $v \in V$, $\sigma(v) \in S$ and $c(v) \in C$ differ for adjacent vertices.

**Definition [6]:** Let $G^S$ be a $S$-valued graph. The vertex chromatic number of $G^S$, denoted by $\chi_S(G^S)$, is defined to be $\chi_S(G^S) = (\min_{v \in V} \sigma(v), \min |C|)$. A coloring $f$ on $G^S$ is said to be $k$-colorable, if it has a proper vertex regular or total proper colorings such that $|C| = k$.

**Definition [4]:** The degree of the vertex $v_i$ of the $S$-valued graph $G^S$ is defined as $deg_S(v_i) = \sum_{(v_i,v_j) \in \Psi} |v_i,v_j|, l$ where $l$ is the number of edges incident with $v_i$.

**Critical Graphs of $S$-Valued Graphs**

In this section, we introduced the concepts of critical $S$-valued graphs.

**Definition:** Consider a $S$-valued graphs $G^S = (V,E,\sigma, \psi)$ the graph $G^S$ is said to be critical if $\chi_S(H) \neq \chi_S(G^S)$ for every proper $S$-valued sub graph $H$ of $G^S$.

Let $G^S = (V,E,\sigma, \psi)$ be such that $\chi_S(G^S)=k$.

From $G^S$ if we remove any vertex $v \in V$ then the graph $H^S = G^S - \{(v,\sigma(v))\}$ is proper sub graph of $G^S$ and also $\chi_S(H^S) = (\min_{v \in V} \sigma(v), k-1) \neq \chi_S(G^S)$ This leads to the following definition.

**Definition:** consider the $S$-valued graph $G^S = (V,E,\sigma, \psi)$ such that $\chi_S(G^S)=k$.

$\chi_S(G^S_{\{v\}})$ for all vertex $v \in V$. This contradicts the definition $G^S$ is a critical $S$-valued graph.

**Proof:** Let $G^S$ be a critical $S$-valued graph, then $\chi_S(G^S) = (\min_{v \in V} \sigma(v), k)$. Therefore $\chi_S(G^S_{\{v\}}) = (\min_{v \in V} \sigma(v), k-1)$.

Therefore $\chi_S(G^S_{\{v\}}) = (\min_{v \in V} \sigma(v), k)$.

Let $v$ be the vertex in $G^S$ such that $v$ does not belong to $V(G_i)$. Then $G^S_{\{v\}}$ is a component of the $S$-valued sub graph $G^S_{\{v\}}$.

**Theorem:** Every critical $S$-valued graph $G^S$ is $S$-connected.

**Proof:** Suppose $G^S$ is not $S$-Connected and $\chi_S(G^S) = (\min_{v \in V} \sigma(v), k)$.

Let $v$ be the vertex in $G^S$ such that $v$ does not belongs to $V(G_i)$. Then $G^S_{\{v\}}$ is a component of the $S$-valued sub graph $G^S_{\{v\}}$.

Therefore $\chi_S(G^S_{\{v\}}) = (\min_{v \in V} \sigma(v), k)$.

This contradicts the definition $G^S$ is a critical $S$-valued graph.

**Theorem:** Every critical $S$-valued graph $G^S$ is $S$-connected.

**Proof:** Let $G^S$ be a critical $S$-valued graph, then $\chi_S(G^S) = (\min_{v \in V} \sigma(v), k)$.

If $G^S$ is not $k$-critical then $\chi_S(G^S_{\{v\}}) = (\min_{v \in V} \sigma(v), k)$ for some vertex $v \in G(V_i)$. Then $G^S_{\{v\}}$ is a component of the $S$-valued sub graph $G^S_{\{v\}}$.

**Theorem:** Every critical $S$-valued graph contains a critical $k$-chromatic $S$-valued graph.

**Proof:** Let $G^S$ be $S$-connected $k$-chromatic graph. Then $\chi_S(G^S) = (\min_{v \in V} \sigma(v), k)$.
Critical Graphs of S-Valued Graphs

\[ x(G\cup\{(v,\sigma(v),(w,\sigma(w))\})=x(G\cup\{(v,\sigma(v))-(w,\sigma(w))\}) = \min_{u\in\{v\cup w\}} \sigma(u), k) \]

If this new S-subgraph is k-critical, then again it is the required subgraph, if not we continue this vertex deletion procedure until we get a k-critical S-valued subgraph.

**Theorem:** Every odd Cycle \( C_{2n+1} \), \( n \geq 1 \) is the only 3 critical S-valued graph

**Proof:** Let \( C_{2n+1} \) be a cycle. We prove this theorem by induction on 2n + 1. Let q=3 start with any vertex say \( v_1 \) in \( C_3 \) assign colour \( c_1 \) to \( v_1 \). Then consider \( N_3[V_1] \). If \( v_2 \in N_3[V_1] \) it should be assigned a colour different from \( c_1 \). Let \( v_2 \) be assigned \( c_2 \). If \( v_2 \in N_3[V_1] \) then \( v_3 \) should be assigned \( c_3 \) different from \( c_1 \) and \( c_2 \). Thus in \( C_3 \), \( x(C_3) = 3 \), clearly \( x(C_3-(v,\sigma(v))) = 2 \), it is not a cycle. Hence \( C_3 \) is a 3 critical. Let us assume the above theorem is holds good for \( C_t \). That is \( C_t \) is t critical, \( t \) is odd.

**Claim:** \( C_{i+2} \) is 3-critical.

Let \( v_i \in C_{i+2} \), then it must be assigned by any one of \( c_i \) colour say \( c_1 \) then consider \( N_3[V_1] \). If \( v_2 \in N_3[V_1] \) and \( v_2 \in C_{i+2} \) then \( v_2 \) must have a colour say \( c_{i+2} \). Since it is of odd cycles, \( c_{i+2} \) it contains at least 2 edges. Both incident at \( v_2 \). Therefore there is another vertex \( v_3 \) which is adjacent to \( v_2 \) and the vertex in \( C_{i+2} \). Therefore \( v_3 \) must assigned a colour \( c_1 \). Clearly \( x(C_{i+2}-(v,\sigma(v))) = 2 \), is not a cycle. Therefore \( C_{i+2} \) is 3-critical. Hence by induction \( C_{i+2} \) is 3 critical for \( i \geq 1 \).

**CONCLUSION**

In this paper, we have discussed the critical graphs for S-valued graphs. Further investigation will be done on colouring of graph products and the critical graphs of S-valued graphs.

**References**


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