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Research Article

INTERACTING DARK FLUIDS IN KALUZA-KLEIN UNIVERSE WITH SPECIAL FORM DECELERATION PARAMETER

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A B S T R A C T

In this paper we deal with Kaluza-Klein universe filled with interacting Dark matter and Holographic dark energy in General Theory of Relativity. The general solutions of Einstein's field equations are obtained by applying the law of special form deceleration parameter proposed by Singha and Debnath (2009).The anisotropy of expansion dies out very quickly and attains isotropy after some finite time. The physical and geometrical parameter of the models are obtained and discussed in detail.

INTRODUCTION

The observations from type Ia Supernova [Perlmutter *et al.* 1998, 1999; Riess *et al.* 2004] and Cosmic Microwave Background [Miller 1999] Radiation indicate that our universe is accelerating. Recent observation PIANK 2013 [Ade 2014] shows that universe consists of 68.3%Dark Energy (DE), 26.8% Dark Matter (DM) and 4.9% baryonic matter. The cause of sudden transition from the earlier deceleration phase to the recent acceleration phase and the source of accelerate expansion are still unknowns.

Many dynamical DE models have been proposed by various researchers such as quintessence with EoS $\omega > -1$ [Barreiro 2000], phantom with EoS $\omega < -1$ [Caldwell 2003], tachyon [Bagla 2003], *k*-essence [Armendariz 2001], dilatonic ghost condensate [Gasperini 2002] and so on.

In the context of the DE problem, though the holographic principle proposes a relation between the holographic dark energy(HDE) density ρ_{DE} and the Hubble parameter H as $\rho_{DE} = H^2$, this is does not contribute to the present accelerated expansion of the universe.

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Granda and Olivers (2009) proposed a holographic density of the form $\rho_{DE} \approx \left(\alpha_1 H^2 + \beta_1 \dot{H}\right)$ where H is the Hubble parameter and α_1, β_1 are constants which must satisfy the restrictions imposed by the current observational data.

Bamba *et al.* (2012) have studied different DE cosmologies (isotropic) in $f(R)$ gravity, $f(R,T)$ gravity, $f(T)$ gravity, Scalar field theory, HDE, coupled DE and *Λ*CDM cosmological models representing the accelerating expansion with the quintessence/phantom nature in details along with Cosmography tests. Many interesting works already have beendone taking HDE model to achieve a good comprehension of the nature of DE [Huang and Li 2004; Gong and Zhang 2005;Hu and Ling 2006; Li *et al.* 2006;Nojiri and Odintsov 2006;Guo *et al.* 2007a, 2007b; Setare 2007a,2007b; Banerjee and Pavon 2007; Zimdahl and Pavón 2007; Granda and Oliveros 2008; Adhav *et. al.* 2014; 2015; Reddy *et al.* 2015].

The theory of five dimensions is due to the idea of Kaluza (1921) and Klein (1926). A five dimensional [*5D*] general relativity is the best outcome of an attempt made by these two by using one extra dimension to unify gravity and electro-magnetism. Realistic unification through the Kaluza-Klein approach requires $d = 5$ manifold topology and the spatial extra dimension radius is of Planck length order. According to Wesson (1984, 1999) and Bellini (2003), the matter is induced in *4D* by *5D* vacuum theory for studying the cosmology of *5D* with pure geometry in non-compact Kaluza-Klein theory. Kaluza-Klein theory is essentially an extension of Einstein's general relativity in five dimensions which is of much interest in particle physics & cosmology.

Singha and Debnath (2009) has defined a special form of deceleration parameter for FRW metric as $q=-\frac{\ddot{a}a}{\dot{a}^2}=-1+\frac{\alpha}{1+a^{\alpha}}$, where $\alpha>0$ is a positive constant and a is mean scale factor of the universe. Adhav *et al.* (2013) extended this law for Bianchi type-I, Bianchi type-V, Bianchi type-III, Bianchi type-VIo and Kantowaski-Sachs space-times respectively.

M. Sharif & F.Khanum (2011) have studied Kaluza–Klein cosmology with modified holographic dark energy, M. Sharif&A. Jawad (2012) studied Modified holographic dark energy in non-flat Kaluza–Klein universe with varying *G*also Sharif&Jawad (2012) studied Interacting modified holographic dark energy in Kaluza-Kleinuniverse.Reddy & Vijaya Lakshmi (2015) have studied Kaluza-Klein Minimally Interacting Holographic Dark Energy Model in a Scalar-sensor Theory of Gravitation and Adhav *et al.*(2015) have Studied Interacting Dark matter and Holographic dark energy in Bianchi type-V universe

Motivated by the above investigations, in this paper, we have studied the Kaluza-Klein universe filled with interacting Dark matter and Holographic dark energy in General Relativity. The general solutions of Einstein's field equations have been obtained by applying the special form deceleration parameter. The anisotropy of expansion dies out very quickly and attains isotropy after some finite time. The physical and geometrical aspects of the models are also discussed.

Metric and Field Equations

We consider a five dimensional Kaluza-Klein metric in the form as

$$ds^2=dt^2-a^2\left(dx^2+dy^2+dz^2\right)-b^2\,d\psi^2, \tag{1}$$

where $a(t)$ and $b(t)$ are the cosmic scale factorsand the fifth coordinate ψ is taken to be space-like.

The Einstein's field equations, in natural limits $(8\pi G=1$ and $c=1)$ are

$$R_{ij}-\frac{1}{2}g_{ij}R=-\left({}^{DM}T_{ij}+{}^{DE}T_{ij}\right), \tag{2}$$

where

$${}^{DM}T_{ij}=\rho_{DM}u_iu_j \quad \text{and}$$

$${}^{DE}T_{ij}=\left(\rho_{DE}+p_{DE}\right)u_iu_j-g_{ij}p_{DE} \tag{3}$$

are energy-momentum tensors for dark matter (pressure less i.e. $\omega_{DM}=0$) and holographic dark energy respectively. Here ρ_{DM} is the energy density of dark matter ρ_{DE} and p_{DE} are the energy density and pressure of holographic dark energy.

In co-moving coordinate systems, the Einstein's field equations (2) for the metric (1) with the help of equations (2) can be written as

$$3\left(\frac{\dot{a}}{a}\right)^2+3\frac{\dot{a}\dot{b}}{ab}=\rho_{DM}+\rho_{DE}, \tag{4}$$

$$2\frac{\ddot{a}}{a}+\frac{\ddot{b}}{b}+\left(\frac{\dot{a}}{a}\right)^2+2\frac{\dot{a}\dot{b}}{ab}=-p_{DE}, \tag{5}$$

$$3\frac{\ddot{a}}{a}+3\left(\frac{\dot{a}}{a}\right)^2=-p_{DE}, \tag{6}$$

where an overhead dot (˙) represents derivative with respect to time *t* .

The volume scale factor V and average scale factor R are given by

$$V=R^4=a^3b. \tag{7}$$

The mean Hubble parameter H is defined as

$$H=\frac{\dot{R}}{R}=\frac{1}{4}\left(H_x+H_y+H_z+H_\psi\right), \tag{8}$$

where $H_x=H_y=H_z=\frac{\dot{a}}{a}$ and $H_\psi=\frac{\dot{b}}{b}$ are the directional Hubble parameters in the directions of x, y z and ψ axes respectively.

The deceleration parameter $q(t)$ is defined by

$$q=-\frac{R\ddot{R}}{\dot{R}^2}=\frac{d}{dt}\left(\frac{1}{H}\right)-1. \tag{9}$$

The mean anisotropy parameter of expansion (Δ) is defined by

$$\Delta=\frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i-H}{H}\right)^2. \tag{10}$$

Subtracting equation (5) from equation (6) and using equation (7), we get

$$\frac{d}{dt}\left(\frac{\dot{a}}{a}-\frac{\dot{b}}{b}\right)+\left(\frac{\dot{a}}{a}-\frac{\dot{b}}{b}\right)\frac{\dot{V}}{V}=0. \tag{11}$$

On integrating equation (11), the scale factors $a(t)$ and $b(t)$ can be written explicitly as

$$a(t)=D_1V^{1/4}\exp\left(X_1\int\frac{dt}{V}\right), \tag{12}$$

$$b(t)=D_2V^{1/4}\exp\left(X_2\int\frac{dt}{V}\right), \tag{13}$$

where the relation $D_1^3D_2=1$ and $3X_1+X_2=0$ are satisfied by D_1, D_2 and X_1, X_2 ;

$$D_1=d^{1/4}, D_2=d^{-3/4}, X_1=\frac{1}{4}X, X_2=-\frac{3}{4}X$$

For the universe, where dark energy and dark matter are interacting to each other the total energy density $(\rho=\rho_{DM}+\rho_{DE})$ satisfies the equation of continuity as

$$\dot{\rho}_{DM}+\dot{\rho}_{DE}+4H(\rho_{DM}+\rho_{DE}+p_{DE})=0. \tag{14}$$

Assuming that the dark matter component is interacting with the dark energy component through an interaction term *Q*, the continuity equation of matter and dark energy can be obtained as

$$\dot{\rho}_{DM} + \left(\frac{\dot{V}}{V}\right)\rho_{DM} = Q \ , \tag{15}$$

$$\dot{\rho}_{DE} + \left(\frac{\dot{V}}{V}\right)(1+\omega_{DE})\rho_{DE} = -Q, \tag{16}$$

where $\omega_{DE} = \frac{p_{DE}}{\rho_{DE}}$ is the equation of state parameter for holographicdark energy and $Q > 0$ measures the strength of the interaction. A vanishing *Q* implies that dark matter and dark energy are separately conserved. In view of continuity equations, the interaction between dark energy and dark matter must be a function of the energy density multiplied by a quantity with units of inverse of time, which can be chosen as the Hubble parameter *H*. There is freedom to choose the form of the energy density, which can be any combination of dark energy and dark matter. Thus, the interaction between dark energy and dark matter could be expressed phenomenologically in the forms as (Guo *et al*.2007a, 2007b; Amendola *et al*. 2007)

$$Q = 4b^2 H\rho_{DM} = b^2 \frac{\dot{V}}{V}\rho_{DM} \ , \tag{17}$$

where b^2 is coupling constant.

Cai and Wang (2005) have taken same relation for interacting dark matter and phantom dark energy in order to avoid the coincidence problem.

Using equations (15) and (17), we get the energy density of dark matter as

$$\rho_{DM} = \rho_0 V^{\left(b^2-1\right)} \quad , \tag{18}$$

where $\rho_0 > 0$ is real constant of integration.

Using equations (17) and (18), we get the interacting term Q as

$$Q = 4\rho_0 b^2 H V^{\left(b^2-1\right)} \ . \tag{19}$$

Cosmological Solutions of Field Equations

In order to obtain the cosmological solutions of the field equations (12) -(13), we assume a mathematical condition which is a special form of deceleration parameter. Singha and Debnath (2009) have defined a special form of deceleration parameter for FRW model as

$$q = -\frac{\ddot{R}R}{\dot{R}^2} = -1 + \frac{\beta}{1+a^{\beta}} \quad , \tag{20}$$

where $\beta > 0$ is a constant and R is mean scale factor of the universe.

Solving Eq. (20), we obtain the mean Hubble parameter H as

$$H = \frac{\dot{R}}{R} = \gamma\,(1+a^{-\beta})\,, \tag{21}$$

where γ is constant of integration.

On integrating Eq. (21), we obtain the mean scale factor as

$$R = V^{1/4} = (e^{\gamma\beta t}-1)^{1/\beta}\,. \tag{22}$$

Using equation (22) for $\gamma = 1$ in the equations (12- 13), we obtain exact value of scale factor as

$$a(t) = D_1 (e^{\beta t}-1)^{1/\beta} \exp\left\{X_1\left[-\frac{1}{4}(e^{\beta t}-1)^{4/\beta} \times (e^{\beta t}-1)^{-4/\beta}\, {}_2F_1\left(\frac{4}{\beta},\frac{4}{\beta};\frac{\beta+4}{\beta};e^{-\beta t}\right)\right]\right\}, \tag{23}$$

$$b(t) = D_2 (e^{\beta t}-1)^{1/\beta} \exp\left\{X_2\left[-\frac{1}{4}(e^{\beta t}-1)^{4/\beta} \times (e^{\beta t}-1)^{-4/\beta}\, {}_2F_1\left(\frac{4}{\beta},\frac{4}{\beta};\frac{\beta+4}{\beta};e^{-\beta t}\right)\right]\right\}, \tag{24}$$

where ${}_2F_1(l,m;n;t)$ is hypergeometric function.

Using equations (23) in equations (18) and (19), we get density of DM and interacting term Q as

$$\rho_{DM} = \rho_0 (e^{\beta t}-1)^{\frac{4(b^2-1)}{\beta}} \quad , \tag{25}$$

$$Q = 4\rho_0 b^2 e^{\beta t}(e^{\beta t}-1)^{\frac{3(b^2-1)}{\beta}-1} \ . \tag{26}$$

Using equations (23) -(24) and equation (25) in the equation (24), we obtain the energy density of HDE as

$$\rho_{DE} = 6e^{2\beta t}(e^{\beta t}-1)^{-2} + 3X(e^{\beta t}-1)^{-8/\beta} - \rho_0 (e^{\beta t}-1)^{\frac{4(b^2-1)}{\beta}} \ . \tag{27}$$

Using equations (25), (27) in the equations (4)-(7), we obtain the pressure of holographic dark energy as

$$p_{DE} = 3X(e^{\beta t}-1)^{-8/\beta} - \frac{3(\beta-2)e^{2\beta t}}{(e^{\beta t}-1)^2} - \frac{3\beta e^{\beta t}}{(e^{\beta t}-1)}\,. \tag{28}$$

The EoS parameter of holographic dark energy is given by

$$\omega_{DE} = \frac{3X(e^{\beta t}-1)^{-8/\beta} - \frac{3(\beta-2)e^{2\beta t}}{(e^{\beta t}-1)^2} - \frac{3\beta e^{\beta t}}{(e^{\beta t}-1)}}{6e^{2\beta t}(e^{\beta t}-1)^{-2} + 3X(e^{\beta t}-1)^{-8/\beta} - \rho_0 (e^{\beta t}-1)^{\frac{4(b^2-1)}{\beta}}}\,. \tag{29}$$

The coincidence parameter r i.e. the ratio of darkenergy densities of dark matter and dark energy is given by

$$r = \frac{\rho_0 (e^{\beta t}-1)^{\frac{4(b^2-1)}{\beta}}}{6e^{2\beta t}(e^{\beta t}-1)^{-2} + 3X(e^{\beta t}-1)^{-8/\beta} - \rho_0 (e^{\beta t}-1)^{\frac{4(b^2-1)}{\beta}}}\,. \tag{30}$$

The mean anisotropy parameter Δ, Hubble parameter H and deceleration parameter q is given as

$$H = e^{\beta t}\left(e^{\beta t}-1\right)^{-1} \quad , \tag{31}$$

$$q = \frac{\beta}{e^{\beta t}} - 1 \quad , \tag{32}$$

$$\Delta = \frac{\left(3X_1^2 + X_1^2\right)\left(e^{\beta t}-1\right)^{\frac{2(\beta-4)}{\beta}}}{4e^{2\beta t}}\,. \tag{33}$$

DISCUSSION

The physical and geometrical behaviors of the cosmological models are as follows:

The deceleration parameter (q)

For the model presented in section-4, the deceleration parameter q varies from $+1$ to -1 as shown in fig.1. It shows that the value of q is positive at the early stage of the universe and becomes negative at late time. For $\beta = 3/2$ the deceleration parameter q is in the range $-1 \le q \le 0.5$ (shaded region in the fig.1) which is consistent with the observations made by Perlmutter *et al.* (1998, 1999) and Riess *et al.* (1998) and the present day universe is undergoing accelerated expansion.

Thus our derived model is suitable to describe the late time evolution of the universe.

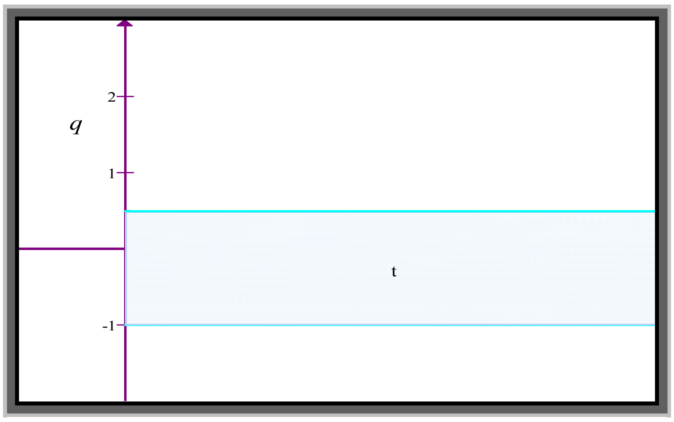

Fig 1 The variation of q vs time (t) for $\beta = 1.5$

The anisotropy parameter of expansion (Δ)

In a Fig. 2 we plot the anisotropy parameter against cosmic time t. It is observed that the anisotropy approaches tozero very quickly. Hence, the models approach to isotropy after some finite time which matches with the recent observationsas the universe is isotropic at large scale. Also Δ diverges as $t \to 0$ and converges to constant as $t \to \infty$

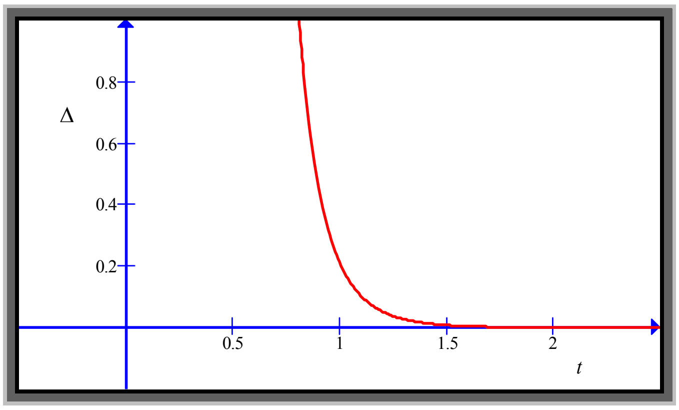

Fig 2 Evolution of anisotropy parameter of expansion (Δ) vs. time (t) for $\beta = 1$ and $(3X_1^2 + X_1^2) = 1$

The equation of state parameter ω_{DE}

Fig. 3 shows that the variation of EoS parameter ω_{DE} starts quintessence region $(\omega_{DE} > -1)$ and after some finite t EoS parameter enters inphantom region $(\omega_{DE} < -1)$. It has been argued that the interacting HDE model can accommodate the transition of the DE equation of state (ω_{DE}) from $(\omega_{DE} > \text{-1} \text{and} \omega_{DE} < -1)$ [Wang *et al.* 2005, 2006].

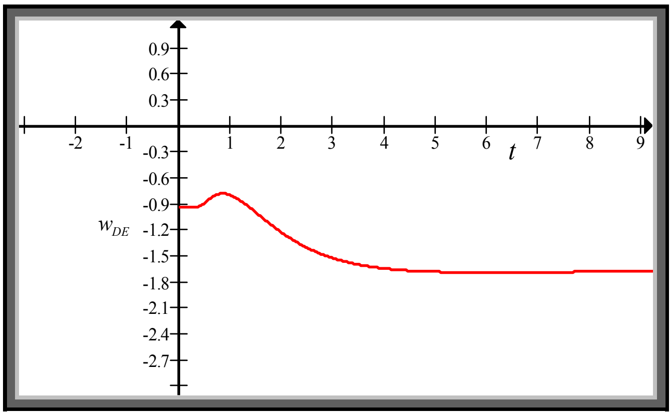

Fig 3 Evolution of EoS parameter (ω_{DE}) vs. time (t) for $k = 1,\ X = 1$ and $\beta = 0.5$

Coincidence parameter

The recent observations demand that the ratio of two energy densities $r = \rho_{DM} / \rho_{DE}$ i.e. the coincidence parameter stays constant or varies very slowly, around the present time, with respect to the universe expansion.

The variation of coincidence parameter r with respect to cosmic time t is as shown in Fig. 4. It is observed that coincidence parameter r at very early stage of evolution varies, but after some finite time it converges to a constant value and remains constant throughout the evolution.

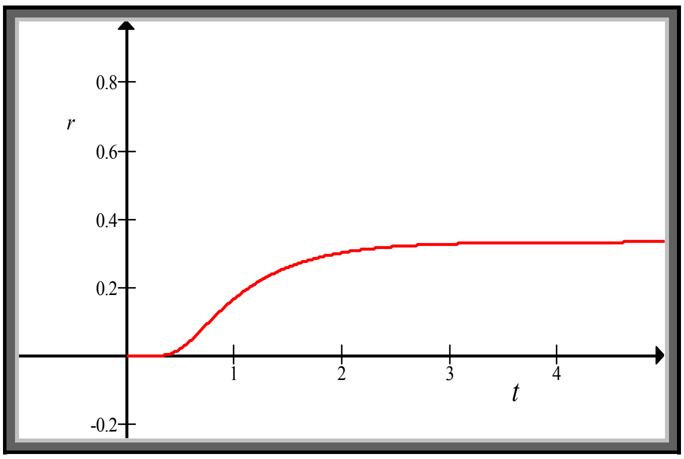

Fig 4 coincidence parameters r vs. cosmic time t

CONCLUSION

Present day universe appears to be isotropic, there is no evidence that the early universe was isotropic. So in this paper we have studied homogeneous and anisotropic kaluza-Klein universe filled with interacting DM and HDE. Our result shows that universe was anisotropic in the early stage and at the late time universe become isotropic.

1. It is observed that the deceleration parameter is a decreasing function and $1 < q < -1$. Therefore the model describe decelerate to accelerate expansion of the universe.
2. The anisotropy parameter for large cosmic time $\Delta \to 0$ .therefore for large cosmic time universe approaches to an isotropic universe. Therefore, the early universe was anisotropy.
3. The value of EoS ω_{DE} starts with quintessence and for large cosmic time EoS parameter enters in the phantom region as $(-1 > \omega_{DE} < -1)$. Which matches the observation with [Wang *et al.* 2005, 2006].
4. One should note that for $b^2 = 0$ in Eq. (17) then it represents the non-interacting Kaluza-Klein model while $b^2 = 1$ yields complete transfer of energy from dark energy to matter. Recently, it is reported that this interaction is observed in the Abell cluster A586 showing a transition of dark energy into dark matter and vice versa for $b^2 = 1$ [Bertolami, O (2007); Jamil, M., Rashid, M. (2008)].

The solution obtained represents interacting dark matter and holographic dark energy model in the five-dimensional space - time. The model exhibits early inflation and late time acceleration which is according to the present scenario of modern cosmology

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