



RESEARCH ARTICLE

**DOMINANCE PREFERENTIAL REFERENCE MARK METHOD WITH ADAPTED CLARKE & WRIGHT HEURISTIC TO SOLVE MULTI-OBJECTIVE VEHICLE ROUTING PROBLEM**

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**ABSTRACT**

The economics heuristic of Clarke & Wright is the principle and reference model for solving the classical vehicle routing problem. In this paper we propose a new version of this method allowing to solve efficiently the multi-objective vehicle routing problem thanks to the preferential reference mark of dominance method. The main result is obtaining the set of efficient solutions  $E(P)$  but in a spread out way. A didactic example validates our step.

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**INTRODUCTION**

The logistics and transports are in the center of many problems met in the industry and in the management of goods and persons. Thus, it is not surprising to see that vehicle routing problems which constitute a facet of the logistics are a part of main problems studied in operations research notably those making part of the combinatorial optimization [1]. The Vehicle Routing Problem (VRP) is a classical combinatorial optimization problem which consists to determine a set of roads for a minimal distance permitting to visit a set of customers from a central deposit (see eg [2], [3], [4], [5] and [6]). In the standard version of this problem, the fleet of vehicles is considered like being homogeneous and unlimited. It is localized in a unique disposal with autonomy allowing to serve all customers. Each vehicle has a total loading capacity of  $C$  units of products and with a limited tour duration to  $D$  units of time. Each customer  $i$  must be visited in order to deliver to him a demand of  $q_i$  units that requires  $s_i$  time of service. The distance  $c_{ij}$  between each couple of localizations  $i$  and  $j$  is known and symmetric. In addition to this, one supposes that the matrix of the distances respects the triangular inequality. The objective of the VRP is to minimize the covered distance by the vehicles in order to deliver the quantities asked by the customers and respecting the constraints of capacity and the different tours duration.

Therefore, many of theoretical stakes but also practical and economic are connected to this family of problems leading the existence of a lot of solving methods. Among these solving methods for Vehicle Routing Problem is the economics heuristic of Clarke and Wright. Its mono-objective version is

one of the first heuristic proposed to solve the Vehicle Routing Problem with constraint of capacity. (see [10], [11], [13]).

The economics heuristic of Clarke and Wright is very simple of application (see [7], [9], [2]). It remains a basic method that knew many variants in which the aim is, on the one hand, to improve it and on the other hand to solve other many kinds of VRP, for example the problem of Vehicles Routing Problem Time Window (VRP/TW). (see [15], [17], [18])

The aim of this work is to hybrid the economics heuristic of Clarke & Wright and the preferential reference mark of dominance method in order to solve the multiobjective vehicle routing problem [19], [20], [21]. This approach finds its interests in one of three approaches identified by Ulungu and Teghem [22], especially the methodological approach, to solve the multiobjective combinatorial optimization problems. Through this didactic example, we prove the performances of this adapted method justified by the good quality of the obtained solutions.

Thus, to present all this, we organized this paper as follows: section 2 presents mathematical formulation of the multiobjective vehicle routing problem and section, 3 present the Preferential reference mark of dominance method, 4 concerns the presentation of Clarke and Wright's economics heuristic principle. As for the multiobjective context of the economics heuristic of Clarke and Wright, it will be presented in section 5. In the following section the didactic example is solved by our adapted method. Finally, we present a

conclusion on observations of E(P) obtainment and some issues of future research.

**Mathematical formulation of multiobjective vehicle routing problem**

Let be considered K objectives functions and v the number of delivery vehicles with a maximal capacity Q intended to serve all customers indicated by the set V from the central deposit during a maximal duration time T. The mathematical formulation of this multiobjective problem of vehicles routing is the following :

$$\begin{aligned}
 (P) \equiv \left\{ \begin{array}{l}
 \text{"min"} \sum_{k=1}^v \sum_{i=1}^n \sum_{j=1}^n c_{ij}^k x_{ijk} \quad k = 1, 2, \dots, m \\
 \sum_{k=1}^v \sum_{j=0}^n x_{ijt} = 1 \quad \forall i \in V \setminus \{0\} \quad (1) \\
 \sum_{i=0}^n x_{ijt} = \sum_{j=0}^n x_{jit} \quad \forall j \in V \setminus \{0\}, \quad t = 1, 2, \dots, v \quad (2) \\
 \sum_{j=1}^n x_{ojt} = 1 \quad t = 1, 2, \dots, v \quad (3) \\
 \sum_{i=0}^n x_{iot} = 1 \quad t = 1, 2, \dots, v \quad (4) \\
 \sum_{i=1}^n \sum_{j=0}^n q_i x_{ijt} \leq Q \quad t = 1, 2, \dots, v \quad (5) \\
 \sum_{i=1}^n \sum_{j=0}^n s_i x_{ijt} + \sum_{i=0}^n \sum_{j=0}^n t_{ij} x_{ijt} \leq T, \quad t = 1, 2, \dots, v \quad (6) \\
 \sum_{i \in U} \sum_{j \in U} x_{ijt} \geq \sum_{j=1}^n x_{ijt}, \quad \forall U \subset V \setminus \{0\}, l \in U; \quad t = 1, 2, \dots, v \quad (7) \\
 x_{ijt} \in \{0, 1\} \quad \forall i, j \in V, \quad i \neq j, \quad t = 1, 2, \dots, v \quad (8)
 \end{array} \right.
 \end{aligned}$$

Interpretation of the different constraints of the vehicle routing problem is:

1. each customer  $i \in V \setminus \{0\}$  is visited one and only one time,
2. each vehicle l arriving at the customer j leaves from there.
3. and
4. each vehicle l leaving the depot comes back to it,
5. respect of the maximal capacity Q of vehicles,
6. respect of the maximal duration time T of routing,
7. elimination of the under-tours to guarantee the connection of the different vehicle routing,
8. precise that it is a combinatorial optimization.

**Solve problem (P) consists to find the entire set or part of the efficient set noted E(P)**

Thus this problem raises from multiobjective combinatorial optimization. It is about to find all or a part of Pareto optimal solutions E(P). It would be illusory to solve it by using exacts methods because of its complexity. In general this complexity is coming from the number of objectives functions and/or

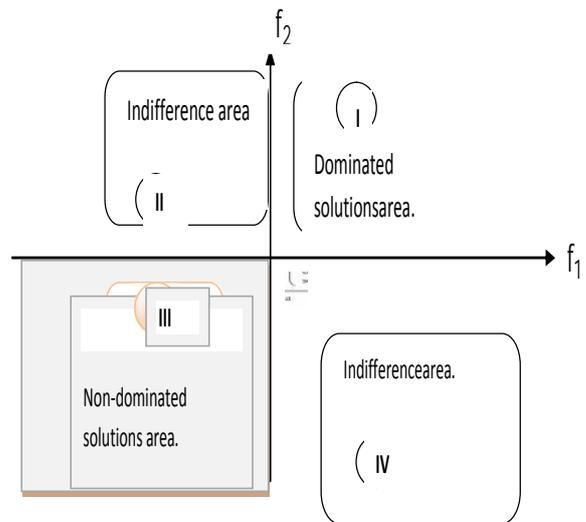
from the constraints and the kind of decision variables. Therefore, the use of a heuristic is required. Even since, it is a good approximation  $\bar{E}(P)$  of E(P) that we must generate.

**Preferential reference mark of dominance method**

Based on preference reference mark to handle dominance notion in feasible solution set of (P). interested reader can consult pages (see [23],[24],[25],[26] and [27]).

**Definitions**

1. Reference mark of dominance of a railable solution a is often referred to an orthonormal reference mark of origin a, dividing the space in four areas of preference in accordance to the diagram of figure 1 below.



**Figure 1** Preferences zones in the dominance relation

2. Let us now consider the objectives space O of a multi-objective combinatorial optimization problem,  $z_1, z_2 \in O$  and  $V(z_1)$  a neighborhood of  $z_1$ . It is said that the solution  $z_2 \in V(z_1)$  certainly improves  $z_1$  if  $z_2$  is situated in the non-dominated solutions area of the preferential reference mark of  $z_1$ . In this case, the acceptance probability of  $z_2$  equals 1. It improves  $z_1$  with an acceptance probability  $\rho$ ,  $0 < \rho < 1$  when it is situated in an indifference area of the  $z_1$  preferential reference mark, and with a nil acceptance probability in the dominated solutions area. In other words, if  $\rho \equiv \mathbb{P}(\text{acceptance of neighborhoods } z_1 \text{ of } z_2)$  then :

$$\rho = \begin{cases} 1 & \text{if } z_2 \in \text{III} \\ 0 < \rho < 1 & \text{if } z_2 \in \text{II} \cup \text{IV} \\ 0 & \text{if } z_2 \in \text{I} \end{cases}$$

3. Let us consider A and B two efficient solutions of a multiobjective combinatorial optimization problem. One says the solution A is more efficient than the solution B if and only if it dominates B in the privileged direction of the decision maker.

The notion of efficiency of a solution is connected to the weights assigned to the objectives defined by the decision maker.

**Stage of Dominance Preferential Reference Mark Method**

Essential of this method is described on the following

references :(Okitonyumbe & al. 2013, 2014 and 2015). Below we present the algorithm stages.

```

Input:
D : Set of admissible solutions
O= F(D) = (fi(a))i=1,...,K, a D.
Output:E(P) : Set of efficient solutions.
Start
E(P)
Represent graphically O
Do while O do
    Choose z in O
    Draw the preferential reference mark of dominance of z
For z' in O \ {z} do
If z' is situated in the non-dominated solutions area then
E(P) ← E(P) ∪ {z'}
O ← O \ {z'}
Endif
If z' is situated in the dominated solutions area then
O ← O \ {z'}
Endif
If the non-dominated solutions area is empty then
E(P) ← E(P) ∪ {z'}
O ← O \ {z'}
Endif
Next
If z' is situated in indifference area then
z ← z'
E(P) ← E(P) ∪ {z'}
O ← O \ {z'}
Endif
Loop
Choose z in E(P)
Draw the preferential reference mark of dominance of z
For z' in E(P) \ {z} do
If z' is situated in the non-dominated solutions area then
E(P) ← E(P) ∪ {z'}
Endif
If z' is situated in the dominated solutions area then
E(P) ← E(P) \ {z'}
Endif
Next
Display E(P)
End
    
```

Please insert: Note that the efficiency of this algorithm was discussed in (Okitonyumbe & al. 2013).

The following section is devoted to outline Clarke & Wright heuristic to be adapted for solving MOCO problem.

**Clarke & Wright economics heuristic**

The economics heuristic of Clarke and Wright is applied when m=1 (classical case). Its principle is based on the calculation of savings realized in uniting two partial routing or two sequences of roads [28], [7], [8]. In the initialization, each customer  $i \in V \setminus \{0\}$  generates a road (O – i – O) joining it by a return journey to the depot. The figure 2 shows it.

From two of these roads, for two customers i and j, it is

elementary to calculate the profit  $\delta_{ij}$  realized in forming only one road (O – i – j – O):

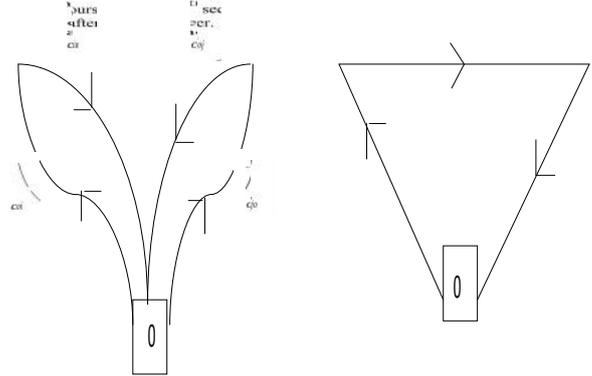
$$\delta_{ij} = c_{i0} + c_{0j} - c_{ij}, \quad i, j \in V \setminus \{0\}$$

This profit remains the same if two roads (O – ... – i – O) and (O – i – ... – O) are merged in the road.

The initial stage of Clarke & Wright method consists to therefore calculate the matrix of saving :

$$\delta_{ij}, \quad i, j \in V \setminus \{0\}$$

Concerning the construction of the routing, two versions are possible : the parallel version that elaborates the simultaneous different tours and the sequential version that constructs the tours one after the other.



**Figure 2** Principe de l'heuristique de Clarke & Wright

In the two versions, once the link established between two customers, it becomes definitive. This heuristic can be considered therefore like a gluttonous heuristic. In particular:

1. In the parallel version, the profits  $\delta_{ij}$  are considered in the decrease order. The first tour (O – i – j – O) that is admissible, that is say verifying the capacity autonomy constraints, ..., is formed.

The process is pursued until the moment when all customers are integrated in one of the formed tours.

2. In the **sequential version**, in the first step, a partial tour (O – i – j – O) is generated on basis of the biggest admissible profit. Only the profits  $\delta_{ki}$  and  $\delta_{jl}$  are considered to prolong the tour either (O – k – i – j – O) or (O – i – j – l – O) with a condition, of course, these tours must remain admissible.

For each iteration, the addition of a customer in first or in last position corresponding to the biggest admissible profit is achieved until the moment when this tour cannot be prolonged without breaking the constraints. In this case, another tour is constructed with the help of the customers not yet affected.

**Remark**

It is noted that, the parallel version often gives better results. But a weakness of the economics heuristic of Clarke and Wright is its propensity : that is the creation of circular roads around the depot. Several attempts for remedying to that have been proposed, notably by modifying the profits :

$$\delta_{ij} \rightarrow \delta'_{ij} = c_{i0} + c_{0j} - \lambda \cdot c_{ij}$$

with help of a parameter  $\lambda$  to fix.

**Adaptation of the economics heuristic of Clarke and Wright to the multiobjective context**

In this section, in conformity with the three proposed approaches by Ulungu and Teghem [22], we have tried to adapt the economics heuristic of Clarke and Wright to the multiobjective context.

**Parameters method adjustment  
Savings calculation**

The Savings are calculated in taking account of each objective by using the following formula :  $s_{ij}^k = c_{i0}^k + c_{0j}^k - c_{ij}^k$ ;  $i, j \in \mathbb{V} \setminus \{0\}; i \neq j, k = 1, \dots, m$ .

The realized profit by joining two segments of roads for two customers  $i$  and  $j$  in only one road  $(O - i - j - O)$  is a line matrix or column matrix of order  $m$  that assigns a value to each objective. It is the same way when one proceeds by a junction of two segments of roads

$(O - \dots - i - O)$  and  $(O - i - \dots - O)$  into a road  $(O - \dots - i - j - \dots - O)$ .

At the initial stage of the methods, one calculates the matrix of the profits that is a tabular in which each compartment is a line matrix or column matrix of order  $m$  (see figure 2 of the paragraph 4). This tabular can be written as following :

$$[s_{ij}^k]; i, j \in \mathbb{V} \setminus \{0\}; i \neq j, k = 1, \dots, m.$$

**Efficient solutions generating**

The profits vector  $(s_{ij}^k; i, j \in \mathbb{V} \setminus \{0\}; i \neq j, k = 1, \dots, m)$  generate a cloud of points in the space. The application of the preference reference mark of dominance, as much time as necessary, produce the sets of efficient solutions by step. These different sets of efficient solutions lead us in the roads construction.

**Roads Construction**

There exists two methods of roads construction by economics heuristic of Clarke and Wright called parallel version and sequential version.

**A. Parallel version**

The profits are considered in order of efficiency in the Pareto sense and the first tour  $(O - i - j - O)$  which is admissible, that is say, verifying the constraints of capacity, of autonomy,...., is formed. The process is pursued until the moment when all customers are integrated in one of the formed tours.

**B. Sequential version**

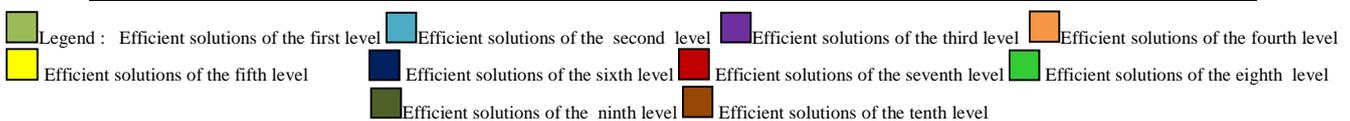
After having generated at the first stage a partial tour  $(O - i - j - O)$  on the basis of efficiency notion of the admissible solutions, only the profits  $s_{ki}^t$  and  $s_{jl}^t$  are considered for prolonging the tour

**Table 1** matrix of distances in kilometers and demands in tons

| N°             | 0 | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11  | 12 | 13 | 14 | 15 |
|----------------|---|----|----|----|----|----|----|----|----|----|----|-----|----|----|----|----|
| 0              | - | 15 | 28 | 30 | 22 | 27 | 21 | 22 | 20 | 36 | 63 | 120 | 22 | 63 | 12 | 27 |
| 1              |   | -  | 21 | 32 | 32 | 41 | 35 | 32 | 22 | 52 | 25 | 37  | 18 | 25 | 22 | 22 |
| 2              |   |    | -  | 18 | 30 | 46 | 47 | 50 | 42 | 45 | 54 | 40  | 54 | 54 | 18 | 20 |
| 3              |   |    |    | -  | 18 | 36 | 43 | 52 | 50 | 24 | 42 | 56  | 49 | 42 | 54 | 36 |
| 4              |   |    |    |    | -  | 18 | 27 | 40 | 43 | 21 | 12 | 43  | 38 | 12 | 49 | 63 |
| 5              |   |    |    |    |    | -  | 16 | 33 | 42 | 15 | 51 | 72  | 55 | 51 | 38 | 37 |
| 6              |   |    |    |    |    |    | -  | 18 | 30 | 32 | 23 | 65  | 52 | 23 | 58 | 40 |
| 7              |   |    |    |    |    |    |    | -  | 15 | 35 | 53 | 37  | 39 | 53 | 55 | 56 |
| 8              |   |    |    |    |    |    |    |    | -  | 28 | 52 | 38  | 64 | 52 | 52 | 43 |
| 9              |   |    |    |    |    |    |    |    |    | -  | 43 | 25  | 42 | 43 | 39 | 72 |
| 10             |   |    |    |    |    |    |    |    |    |    | -  | 40  | 53 | 35 | 64 | 65 |
| 11             |   |    |    |    |    |    |    |    |    |    |    | -   | 62 | 26 | 42 | 37 |
| 12             |   |    |    |    |    |    |    |    |    |    |    |     | -  | 33 | 53 | 38 |
| 13             |   |    |    |    |    |    |    |    |    |    |    |     |    | -  | 62 | 25 |
| 14             |   |    |    |    |    |    |    |    |    |    |    |     |    |    | -  | 40 |
| d <sup>t</sup> | - | 3  | 3  | 4  | 2  | 4  | 2  | 3  | 4  | 5  | 3  | 4   | 2  | 5  | 4  | 3  |

**Table 2** Cloud points representing savings in distance and priority

|    | 2       | 3       | 4       | 5       | 6       | 7       | 8       | 9       | 10      | 11       | 12       | 13      | 14       | 15       |
|----|---------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|---------|----------|----------|
| 1  | (22,29) | (13,28) | (5,27)  | (1,26)  | (1,25)  | (5,24)  | (13,23) | (-1,22) | (53,21) | (98,20)  | (19,19)  | (53,18) | (5,17)   | (20,16)  |
| 2  |         | (40,27) | (20,26) | (9,25)  | (2,24)  | (0,23)  | (6,22)  | (14,21) | (47,20) | (108,19) | (-4,18)  | (47,17) | (22,16)  | (35,15)  |
| 3  |         |         | (34,25) | (21,24) | (8,23)  | (0,22)  | (0,21)  | (42,20) | (51,19) | (94,18)  | (3,17)   | (51,16) | (-12,15) | (21,14)  |
| 4  |         |         |         | (31,23) | (16,22) | (4,21)  | (-1,20) | (37,19) | (73,18) | (99,17)  | (6,16)   | (73,15) | (-15,14) | (-14,13) |
| 5  |         |         |         |         | (32,21) | (16,20) | (5,19)  | (48,18) | (39,17) | (75,16)  | (-6,15)  | (39,14) | (1,13)   | (17,12)  |
| 6  |         |         |         |         |         | (25,19) | (11,18) | (25,17) | (61,16) | (76,15)  | (-9,14)  | (61,13) | (25,12)  | (8,11)   |
| 7  |         |         |         |         |         |         | (27,17) | (23,16) | (32,15) | (105,14) | (5,13)   | (32,12) | (-21,11) | (-7,10)  |
| 8  |         |         |         |         |         |         |         | (28,15) | (31,14) | (102,13) | (-22,12) | (31,11) | (-20,10) | (4,9)    |
| 9  |         |         |         |         |         |         |         |         | (56,13) | (131,12) | (16,11)  | (56,10) | (-1,9)   | (-9,8)   |
| 10 |         |         |         |         |         |         |         |         |         | (143,11) | (32,10)  | (91,9)  | (11,8)   | (25,7)   |
| 11 |         |         |         |         |         |         |         |         |         |          | (80,9)   | (157,8) | (90,7)   | (110,6)  |
| 12 |         |         |         |         |         |         |         |         |         |          |          | (52,7)  | (-19,6)  | (11,5)   |
| 13 |         |         |         |         |         |         |         |         |         |          |          |         | (13,5)   | (65,4)   |
| 14 |         |         |         |         |         |         |         |         |         |          |          |         |          | (-1,3)   |



either  $(O - k - i - j - O)$  or  $(O - i - j - l - O)$ , on condition of course these tours remain admissible.

For each iteration, the addition of a customer in first or in last position corresponding to the biggest admissible profit

achieved until the moment when this tour can no longer be prolonged without breaking the constraints. In this case, another tour is constructed with the help of the customers not yet affected, on the basis of efficiency of savings.

**Definition**

Let be considered A and B two efficient solutions of a multiobjective combinatorial optimization problem, one says that the solution A is more efficient than the solution B if and only it has a bigger value that B on the privileged direction given by the decision-maker. The efficiency solution notion be bound by weights assigned to the objectives by the decision-maker.

**Didactic example**

A pharmaceutical industry wants to test a new product on the market. It possesses a warehouse and a fleet of vehicles with a maximum delivery capacity of 8 tons a vehicle. The demands of 15 customers are known (see tabular 1). The distances between the customers are symmetrical and verify the triangular inequality. The customers are classified according to the decrease order of priority that is encoded of 1 to 15.

**Decision maker concerns**

The different decision-maker concerns for organizing of the distribution routing are:

1. minimized the covered distances,
2. minimized the fleet size,
3. maximized the customers priority.

The cost is 25 UM per kilometer covered and the fixed cost of a vehicle amounts to 2500 UM.

**Cloud points representing savings in distance and priority between two customers to join**

In a space with two dimensions, respectively the profit in distance and in priority, we present the coordinates of partial tours visiting two customers. The dimension "size of the fleet" to minimize will intervene in roads building while respecting the capacity constraints of vehicles.

**Efficient solutions**

Let us note that  $E_1(P)$ , the set of efficient solutions obtained at the stage 1, thus we have: Initially for  $l=1$ , the set of solutions potentially efficient generated by the Preferential reference mark of dominance method is:  $E_1(P)=\{(22,29), (40,27), (53,21), (98,20), (108,19), (143,11)\}$  corresponding respectively to the following capacity  $\{6,7,6,7,7,7\}$

After suppressing of the efficient solutions of first stage, we obtain at the second stage:

$$E_2(P)=\{(105,14),(99,17),(94,18),(110,6),(13,28),(20,26),(34,25),(51,19),(47,20)\}$$

$$E_3(P)=\{(10,13),(76,15),(75,16),(73,18),(5,27),(9,25),(21,24),(31,23),(32,21)\}$$

$$E_4(P)=\{(91,9),(61,16),(53,18),(73,15),(37,19),(16,22),(8,23),(5,24),(1,26)\}$$

$$E_5(P)=\{(89,7),(80,9),(61,13),(25,19),(14,21),(13,23),(16,20),(2,24),(1,25)\}$$

**Table 3** Roads construction by the parallel version

| Solution | Capacity | Distance | Tour                           |
|----------|----------|----------|--------------------------------|
| (143,11) | 7        | 223Km    | (0-10-11-0)                    |
| (108,19) | 10       |          | (0-10-11-2-0) Not allowable    |
| (98,20)  | 7        | 76Km     | (0-2-3-0)                      |
| (53,21)  | 10       |          | (0-10-11-1-0) Not allowable    |
| (40,27)  | 10       |          | (0-1-10-11-0) Not allowable    |
| (22,29)  | 10       |          | (0-1-2-3-0) Not allowable      |
| (105,14) | 10       |          | (0-10-11-7-0) Not allowable    |
| (99,17)  | 9        |          | (0-10-11-4-0) Not allowable    |
| (94,18)  | 11       |          | (0-10-11-3-0) Not allowable    |
| (110,6)  | 10       |          | (0-10-11-15-0) Not allowable   |
| (13,28)  | 10       |          | (0-2-3-1-0) Not allowable      |
| (20,26)  | 9        |          | (0-4-2-3-0) Not allowable      |
| (34,25)  | 9        |          | (0-2-3-4-0) Not allowable      |
| (51,19)  | 10       |          | (0-2-3-10-0) Not allowable     |
| (47,20)  | 10       |          | (0-10-2-3-0) Not allowable     |
| (102,13) | 11       |          | (0-10-11-8-0) Not allowable    |
| (76,15)  | 9        |          | (0-10-11-6-0) Not allowable    |
| (75,16)  | 11       |          | (0-10-11-5-0) Not allowable    |
| (73,18)  | 9        |          | (0-4-10-11-0) Not allowable    |
| (5,27)   | 5        | 69Km     | (0-1-4-0)                      |
| (9,25)   | 11       |          | (0-5-2-3-0) Not allowable      |
| (21,24)  | 11       |          | (0-2-3-5-0) Not allowable      |
| (31,23)  | 9        |          | (0-1-4-5-0) Not allowable      |
| (32,21)  | 6        | 64Km     | (0-5-6-0)                      |
| (91,9)   | 12       |          | (0-13-10-11-0) Not allowable   |
| (61,16)  | 9        |          | (0-6-10-11-0) Not allowable    |
| (53,18)  | 9        |          | (0-14-1-4-0) Not allowable     |
| (73,15)  | 10       |          | (0-1-4-13-0) Not allowable     |
| (37,19)  | 10       |          | (0-1-4-9-0) Not allowable      |
| (16,22)  | 12       |          | (0-1-4-6-5-0) Not allowable    |
| (8,23)   | 13       |          | (0-2-3-6-5-0) Not allowable    |
| (5,24)   | 10       |          | (0-7-1-4-0) Not allowable      |
| (1,26)   | 9        |          | (0-5-1-4-0) Not allowable      |
| (90,7)   | 11       |          | (0-10-11-14-0) Not allowable   |
| (61,13)  | 11       |          | (0-5-6-13-0) Not allowable     |
| (80,9)   | 9        |          | (0-10-11-12-0) Not allowable   |
| (14,21)  | 12       |          | (0-9-2-3-0) Not allowable      |
| (14,23)  | 9        |          | (0-8-1-4-0) Not allowable      |
| (25,19)  | 9        |          | (0-5-6-7-0) Not allowable      |
| (16,20)  | 9        |          | (0-7-5-6-0) Not allowable      |
| (2,24)   | 9        |          | (0-6-2-3-0) Not allowable      |
| (65,4)   | 8        | 115Km    | (0-13-15-0)                    |
| (56,13)  | 9        |          | (0-9-10-11-0) Not allowable    |
| (47,17)  | 12       |          | (0-13-2-3-0) Not allowable     |
| (19,19)  | 7        | 94Km     | (0-12-1-4-0)                   |
| (11,18)  | 10       |          | (0-5-6-8-0) Not allowable      |
| (6,22)   | 11       |          | (0-8-2-3-0) Not allowable      |
| (39,17)  | 9        |          | (0-10-5-6-0) Not allowable     |
| (4,21)   | 10       |          | (0-6-1-4-7-0) Not allowable    |
| (52,7)   | 10       |          | (0-12-13-15-0) Not allowable   |
| (27,17)  | 7        | 57Km     | (0-7-8-0)                      |
| (35,15)  | 15       |          | (0-13-15-2-3-0) Not allowable  |
| (32,15)  | 14       |          | (0-8-7-10-11-0) Not allowable  |
| (25,17)  | 11       |          | (0-5-6-9-0) Not allowable      |
| (5,19)   | 13       |          | (0-6-5-8-7-0) Not allowable    |
| (23,16)  | 12       |          | (0-8-7-9-0) Not allowable      |
| (31,14)  | 10       |          | (0-10-8-7-0) Not allowable     |
| (32,12)  | 12       |          | (0-8-7-13-0) Not allowable     |
| (5,17)   | 9        |          | (0-14-1-4-0) Not allowable     |
| (32,10)  | 12       |          | (0-8-7-9-0) Not allowable      |
| (22,16)  | 11       |          | (0-14-2-3-0) Not allowable     |
| (3,17)   | 14       |          | (0-2-3-12-1-4-0) Not allowable |
| (25,12)  | 10       |          | (0-5-6-14-0) Not allowable     |
| -        | 5        | 72Km     | (0-9-0)                        |
| -        | 4        | 24Km     | (0-14-0)                       |

- $E_6(P) = \{(65,4), (56,13), (37,17), (19,19), (6,22)\}$
- $E_7(P) = \{(56,10), (39,17), (4,21), (11,18)\}$
- $E_8(P) = \{(52,7), (35,15), (27,17)\}$
- $E_9(P) = \{(32,15), (25,17), (5,19)\}$
- $E_{10}(P) = \{(23,16), (31,14), (32,12), (5,17)\}$
- $E_{11}(P) = \{(32,10), (22,16), (3,17), (25,12)\}$

Now, we can go along to the construction of roads

**Roads construction**

**Roads construction in Parallel method**

After the execution of the three stages:  $E_8(P)$ ,  $E_9(P)$ ,  $E_{10}$  without success, we are able to notice that it remains two customers 9 and 14 when the tour (0-9-14-0) isn't allowable because it breaks the capacity constraint of the vehicles. Whence the construction of two elementary tours (0-9-0) and (0-14-0). Then, the solution obtained by the parallel version is (732,120,8) composed of eight tours : (0-10-11-0), (0-2-3-0), (0-5-6-0), (0,13-15-0), (0-12-1-4-0), (0-7-8-0), (0-9-0) and (0-14-0) and covering a total distance of 732 km with a total priority of 120.

**Sequential method**

**Table 4**

| Solution | Capacity | Distance | Tour         |
|----------|----------|----------|--------------|
| (143,11) | 7        | 223Km    | (0-10-11-0)  |
| (98,20)  | 7        | 64Km     | (0-2-3-0)    |
| (65,4)   | 8        | 115KM    | (0-13-15-0)  |
| (32,21)  | 6        |          | (0-5-6-0)    |
| (-6,15)  | 8        | 114Km    | (0-12-5-6-0) |
| (5,27)   | 5        |          | (0-1-4-0)    |
| (5,24)   | 8        | 108Km    | (0-7-1-4-0)  |
| (-20,10) | 8        | 84Km     | (0-8-14-0)   |
| -        | 5        | 72Km     | (0-9-0)      |

The obtained solutions by the sequential version are composed of seven tours : (0-10-11-0), (0-2-3-0), (0,13-15-0), (0-12-5-6-0), (0-7-1-4-0), (0-8-14-0) and (0-9-0) whence the global cost is 37000UM and the covered distance is 780km with total priority of 120.

With a similar reasoning, by initializing the round with the solutions: (108,19), (98,20), (53,21), (40, 27) and (22,29) we obtain respectively the following solutions :

1. Solution: (833, 120,8) corresponding to the rounds: (0-2-11-0), (0-1-10-0), (0-2-3-0), (0-7-12-15-0), (0-5-6-0), (0-8-14-0), (0-9-0) and (0-13-0) .
2. Solution: (745, 120,8) conforms to the rounds: (0-1-11-0), (0-2-3-0), (0-4-5-0), (0-12-13-0), (0-7-6-10-0), (0-8-15-0), (0-9-0) and (0-14-0).
3. Solution: (754, 120,8) appropriate with the rounds: (0-1-10-0), (0-2-3-0), (0-4-11-0), (0-5-6-0), (0-7-8-0), (0-12-13-0), (0-14-15-0) and (0-9-0).
4. Solution: (754, 120, 8) corresponding with the Rounds: (0-2-3-0), (0-1-10-0), (0-4-11-0), (0-5-6-0), (0-7-8-0), (0-12-13-0), (0-14-15-0) and (0-9-0).
5. Solution: (862, 120, 8) conforms to the rounds: (0-1-2-0), (0-10-11-0), (0-5-6-0), (0-7-8-0), (0-12-13-0), (0-14-15-0) and (0-9-0).

**RESULTS AND DISCUSSION**

The obtained solutions by the sequential method are : (780,120,7), (833,120,8), (745,120,8), (754,120,8) and (862,120,8) but the decision maker choice must be worked on two solutions : (780,120,7) and (745,120,8) because the other are dominated. If it were asked to us to give a point of view to the decision maker, it is the first solution which we would have advised to choose because the additional cost generated by the increase of the distance that is 35 Km X 25 Um/Km = 875 Um is negligible compared to the fixed cost of a vehicle which amounts to 2500 UM.

**CONCLUSION AND PERSPECTIVES**

As the most used of heuristics in multiobjective optimization, the economics heuristic of Clark and Wright is dedicated to mono-objective optimization. This work is the best adaptation of this heuristic to multiobjective optimization. Through the didactic example of the results obtained above we see that this adaptive method is the best method to solve the multiobjective vehicles routing problems with an unlimited fleet and unique depot. Besides we have proved, in this paper that, the sequential version gave the better result than the parallel version in the multiobjective context, that is the contrary in mono-objective.

It is also necessary to add the fact that the sequential version permitted us to get these solutions in nine iterations whereas the parallel version drove us until sixty-five iterations creating thus several circular roads around the depot. Indeed, it reduced the size of the fleet to seven vehicles while keeping the same total priority than the parallel version. In the future works we will try to program the sequential version of this multiobjective version of economics heuristic of Clarke and Wright in order to solve lots of problems.

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